Momentum - Center of Mass

Note Title 10/11/2010

Demo Progression:

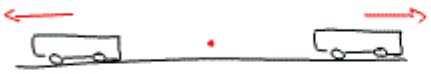
- COM of single cart on track 
- COM of two stationary carts

$x_{cm} = 100$

- Velocity of COM of two moving carts before and after collision

Before 

$v_{cm} = 0$

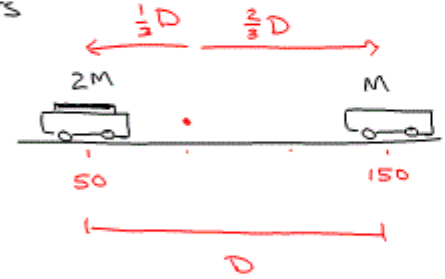
After 

$v_{cm} = 0$

$v_{cm_i} = v_{cm_f}$

Explanation: Newton's 1st Law holds not just for single particles, but also for systems of particles!

- COM of two stationary carts of unequal mass



$x_{cm} = 83.3$

Position of COM

For 2 objects: $x_{cm} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$

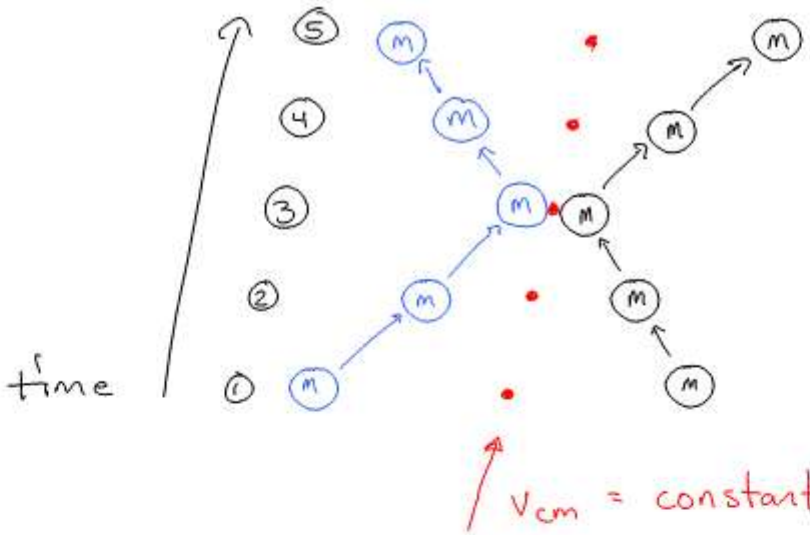
For n objects: $x_{cm} = \frac{\Sigma x_i m_i}{\Sigma m}$

Velocity of COM

$v_{cm} = \frac{v_1 m_1 + v_2 m_2}{m_1 + m_2}$

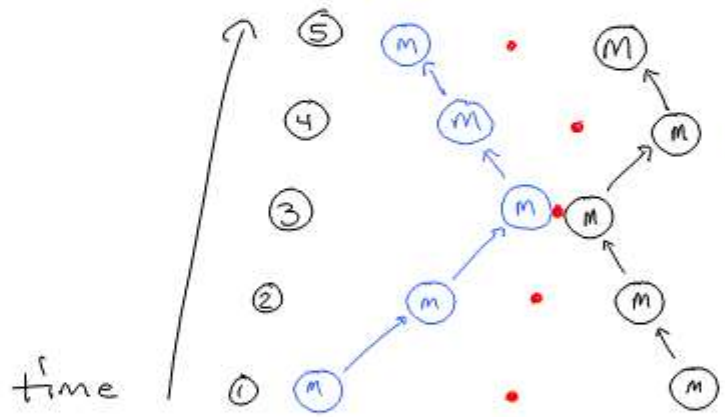
$v_{cm} = \frac{\Sigma x_i m_i}{\Sigma n}$

- COM of a 2D collision between 2 equal masses



v_{cm} = constant!

- COM of 2D collision, plus an external force applied to M_2 at time $t=4$:



Mass 2 is kicked to the left

v_{cm} = changes at $t=4s$

- Fireworks

Note the parabolic trajectory of the firework's COM both before and after the explosion!

For a uniform mass distribution:

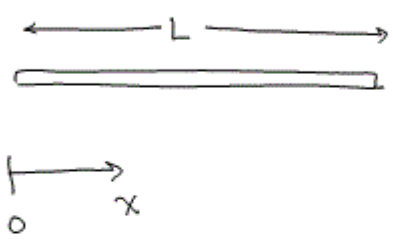
$$x_{cm} = \frac{\int m\,dx}{M}$$

where m = mass as a function of position

dx = a small piece of position

M = total mass

Example: Show that the COM of a meterstick is at its halfway point.



Model the meterstick as a uniform mass (M) distributed over a total length (L).

Therefore, a small piece (dx) has a small mass (dm) given by

$$\frac{dm}{dx} = \frac{M}{L}$$

This quantity $\frac{dm}{dx}$ is called "linear density":

$$\rho = \frac{M}{L}$$

The mass of a piece of length x is therefore $m = \frac{M}{L}x$

The COM is given by

$$x_{cm} = \frac{\int \rho\,dx}{M} = \frac{\int_0^L \frac{M}{L}x\,dx}{M}$$

$$x_{cm} = \frac{\frac{M}{L}\int_0^L x\,dx}{M}$$

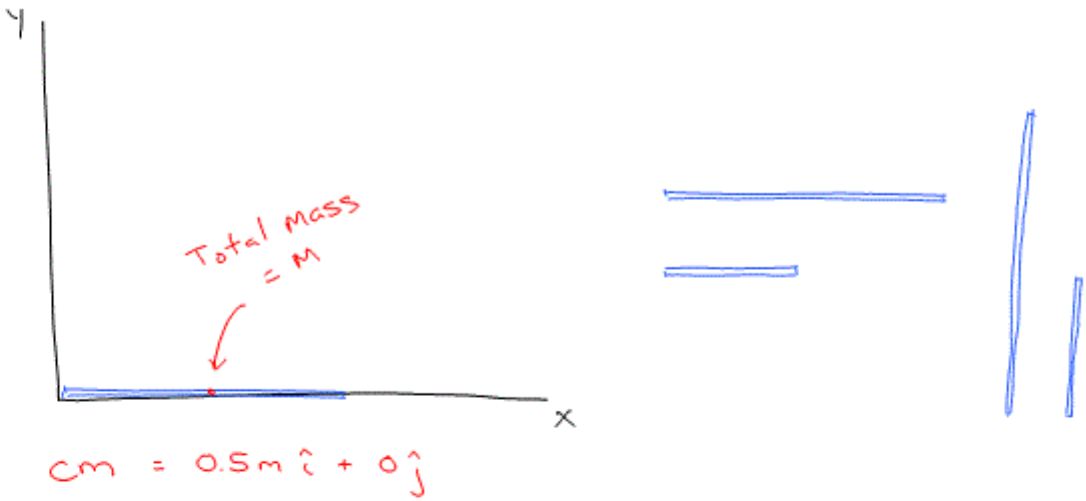
$$x_{cm} = \frac{1}{L}\left(\tfrac{1}{2}x^2\right)\Big|_0^L = \frac{1}{L}\left(\tfrac{1}{2}L^2\right) - \frac{1}{L}\left(\tfrac{1}{2}(0)^2\right)$$

$$\boxed{x_{cm} = \frac{L}{2}}$$

$\therefore$ Exactly as predicted!

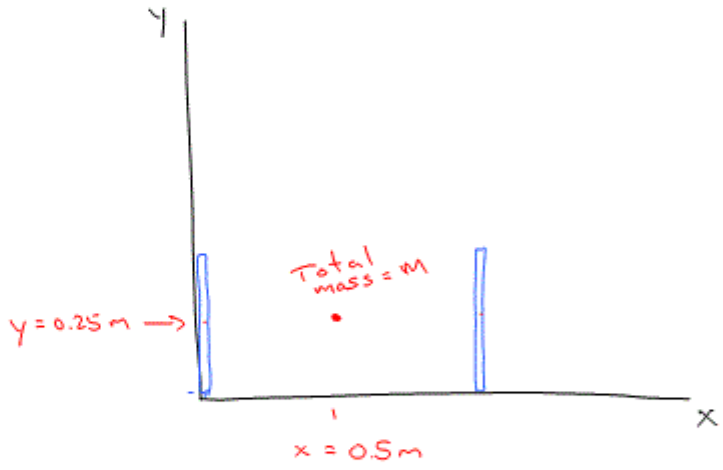
Demo Progression (continued)

- COM of a single meterstick on an x-y plane



$cm = 0.5m\,\hat{i} + 0\,\hat{j}$

- COM of 2 x ½ metersticks oriented vertically on x-y plane



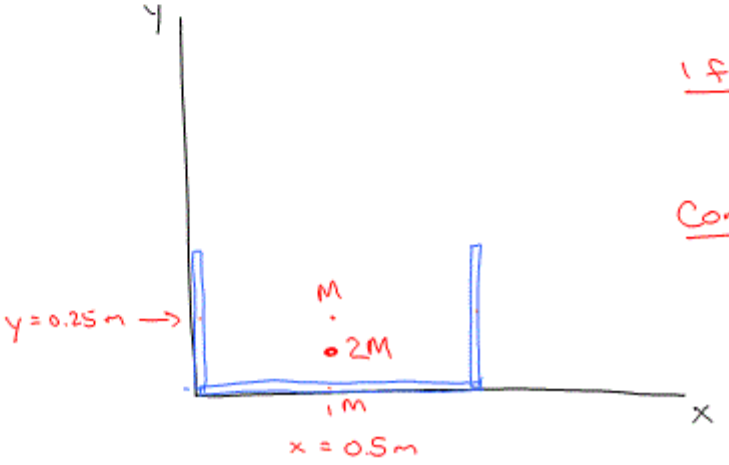
$cm = 0.5m\,\hat{i} + 0.25m\,\hat{j}$

- Combined COM of 1 full + 2 x ½ metersticks

2 x ½ metersticks
M at (0.5, 0.25)

1 full meterstick
M at (0.5, 0)

Combined
2M at (0.5, 0.125)



$cm = 0.5m\,\hat{i} + 0.125m\,\hat{j}$