

# AP Physics - Momentum - Proof of Cons of $\vec{P}$ + Examples

Note Title

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Proof of Conservation of Momentum with Newton's 3rd Law:

Consider two bodies, ① and ②. Newton's 3rd Law states that it is impossible for ① to exert a force  $F_{1 \rightarrow 2}$  on ② without ② exerting an equal and opposite force  $F_{2 \rightarrow 1}$  on ①.

$$\vec{F}_{1 \rightarrow 2} = -\vec{F}_{2 \rightarrow 1}$$

Moreover, it would be impossible for either force to exist for any length of time longer than the other. Therefore:

$$\vec{F}_{1 \rightarrow 2} t = -\vec{F}_{2 \rightarrow 1} t$$

Impulse-Momentum theorem says that an impulse  $Ft$  causes a change in momentum  $\Delta p$ , such that

$$\Delta \vec{p}_2 = -\Delta \vec{p}_1$$

or

$$\vec{p}_{2f} - \vec{p}_{2i} = -(\vec{p}_{1f} - \vec{p}_{1i})$$

Rearranging the above, putting initial momentums on the left and final momentums on the right, we have

$$-(\vec{P}_{2f} - \vec{P}_{2i}) = \vec{P}_{1f} - \vec{P}_{1i}$$

$$\vec{P}_{2i} - \vec{P}_{2f} = \vec{P}_{1f} - \vec{P}_{1i}$$

$$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}$$

$$\Sigma \vec{P}_i = \Sigma \vec{P}_f$$

$\therefore$  We have proved that momentum is conserved!

Example: What is the average recoil force for a machine gun that shoots 55g bullets at 1050 m/s at a rate of 3 bullets/seconds?

$$Ft = \Delta p$$

$$Ft = mv_f - 0$$

$$F = \frac{mv}{t}$$

$$F = \frac{(0.055 \text{ kg})(1050 \text{ m/s})}{(\frac{1}{3} \text{ s})}$$

$$F = 173 \text{ N}$$