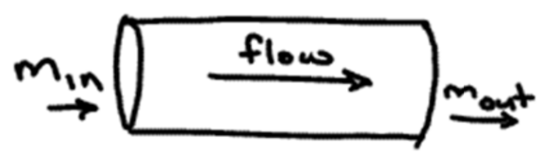
AP Physics - Fluids - Fluid Dynamics

Note Title 11/20/2007

Fluid Continuity Equation

Consider a large pipe full of water. Assuming that water can't be compressed (incompressible), we know that conservation of mass says that if 10 kg of water enters one side, 10 kg of water must leave through the other side.



$$m_{in} = m_{out}$$

In a given amount of time t:

$$\frac{m_{in}}{t} = \frac{m_{out}}{t}$$

Also, since water is incompressible, we know it has a constant density, and since $m = \rho V$

$$\frac{\cancel{\rho} V_{in}}{t} = \frac{\cancel{\rho} V_{out}}{t}$$

$$\frac{V_{in}}{t} = \frac{V_{out}}{t}$$

If the pipe is cylindrical, then it has a constant area, and the volume $V = A \cdot l$

$$\frac{V_{in}}{t} = \frac{V_{out}}{t}$$

$$A_{in} \cdot \frac{l}{t} = A_{out} \cdot \frac{l}{t}$$

Given that speed $v = \frac{l}{t}$, we conclude that

Flow continuity rule (comes directly from conservation of mass)

$$\boxed{A_{in} \cdot v_{in} = A_{out} \cdot v_{out} \quad \text{-or-} \quad \frac{A_{in}}{A_{out}} = \frac{v_{out}}{v_{in}}}$$

$\therefore$ The speed of an incompressible fluid in a pipe is inversely proportional to the ratio of cross-sectional area.

As a pipe gets fatter, speed slows.

As a pipe gets skinner, flow gets faster.

Bernoulli's Equation

If we apply flow continuity (conservation of mass) and conservation of energy to Pascal's Law, we can derive the following equation:

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

See your book for the derivation!

Pressure at a given point

Density

Velocity

Density

$9.8\,m/s^2$

Height of point in fluid from some reference height

—or—

$$P_A + \frac{1}{2}\rho v_A^2 + \rho g h_A = P_B + \frac{1}{2}\rho v_B^2 + \rho g h_B$$

Example: A constricting pipe

Cancel, since they are equal

$\rho g h_A = \rho g h_B$

$$P_A + \frac{1}{2}\rho v_A^2 + \cancel{\rho g h_A} = P_B + \frac{1}{2}\rho v_B^2 + \cancel{\rho g h_B}$$