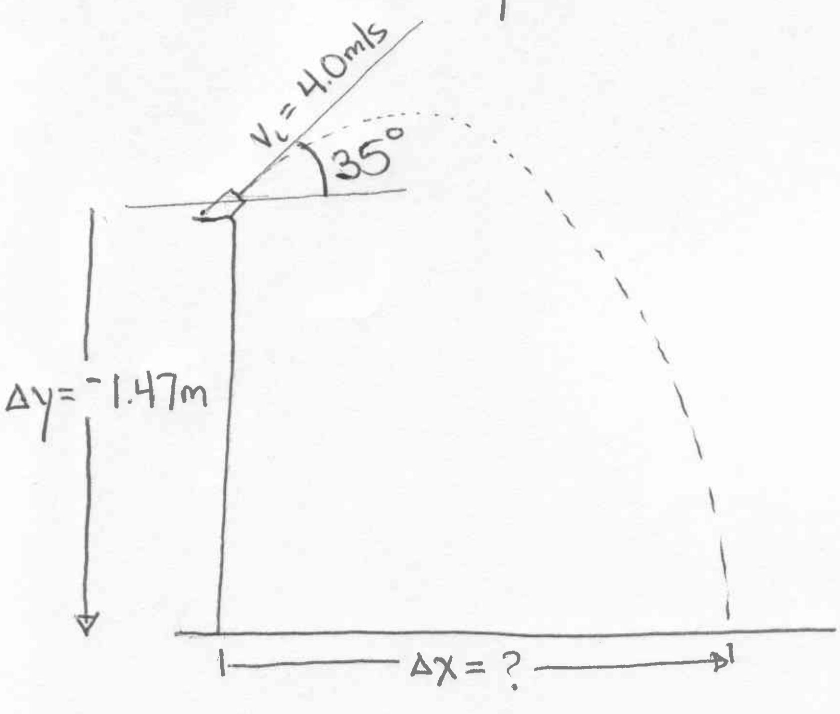
AP Physics - 2D Motion

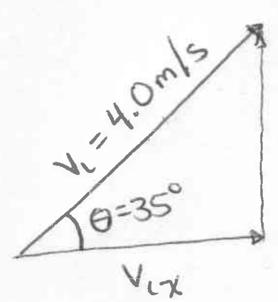
So far in this unit we have learned how to do two things: manipulate vectors, and solve projectile problems with either horizontal or vertical launches. Our final task is to put these two together to form...

- Launching projectiles at an angle, with a landing height different than starting height (aka. "angle off a cliff", "siege cannon", etc.)

Example: How far will our projectile launcher shoot with an initial height of 1.47m, a launch velocity of 4.0 m/s, and a firing angle of 35°?



Step 0 - Find v_{ix} and v_{iy}:



$\sin 35° = \frac{v_{iy}}{4.0}$

$v_{iy} = 4.0 \sin 35°$

$\boxed{v_{iy} = 2.29 \text{ m/s}}$

$\cos 35° = \frac{v_{ix}}{4.0}$

$\boxed{v_{ix} = 3.28 \text{ m/s}}$ *

Angle off a cliff (continued)

* Note that since gravity only affects the projectile vertically, and there are no other forces to affect things horizontally, there is no horizontal acceleration. Therefore:

$$v_{ix} = v_{fx} = \bar{v}_x = 3.28 \text{ m/s}$$

Now we can start filling out our table with givens and unknowns:

x

G: $\bar{v}_x = 3.28$ m/s

Step 3 — $t = 0.829$ s

U: $\Delta x = ?$

E: $\bar{v}_x = \frac{\Delta x}{\Delta t}$

$\Delta x = \bar{v}_x \Delta t$

Step 4 — $\Delta x = (3.28)(0.829)$

$\boxed{\Delta x = 2.72 \text{ m}}$

∴ The projectile will land 2.72m from the launcher.

y

step 1

G: $v_{iy} = 2.29$ m/s

$a_y = -9.8$ m/s

$\Delta y = -1.47$ m

U: t

E: $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$

$0 = \frac{1}{2}a_y t^2 + v_{iy}t - \Delta y$

...using Quadratic Formula to solve for time (t)... step 2

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-v_{iy} \pm \sqrt{v_{iy}^2 - 4(\frac{1}{2}a_y)(-\Delta y)}}{a_y}$$

$$t = \frac{-2.29 \pm \sqrt{2.29^2 - 4(\frac{-9.8}{2})(+1.47)}}{-9.8}$$

$t = 0.234 \pm 0.595$

$t = 0.829$ s

There are numerous variations on this problem, for example: "You are about to use a catapult to launch fireballs over a wall. Your catapult launches projectiles at a 45° angle at 100 m/s. The walls are 20m high. How far back from the wall should you position the catapult?"

Givens: θ, v_i, Δy, a_y Unknown: Δx

"If archers necessitate you standing back 50 yds from the wall, what is the maximum angle at which you can launch fireballs to have them travel over the wall?"

Givens: v_i, Δy, Δx, a_y Unknown: θ

And so on....

One last question: Assuming no air friction and level ground, at what (launch) angle will a projectile travel the farthest? Answer: 45°

Why? → Range is given by the equation $\Delta x = \bar{v}_x \Delta t$:

90° – at 90° (vertical), flight time is maximized, but $\bar{v}_x = 0$

0° – at 0° (horizontal), $\bar{v}_x$ is maximized, but $\Delta t = 0$

45° – Neither $\bar{v}_x$ nor Δt are maximum, but their product is.