Unit 2 - 2D Motion - Day 1 8/5/07

Scalar ≡ A physical quantity that only has magnitude (size or amount), but no direction.

Scalar Examples:

- 10 apples
- 5 meters
- 2 kilograms
- 47 mph
- 14.7 pounds per square inch

Vector ≡ A physical quantity that has both magnitude and direction.

Vector Examples:

- 5 meters up
- 4.4 pounds (force) down
- 47 mph North
- 5 m/s^2 at 20° E of N

In this unit, we will use the following quantities:

	scalar form	vector form
meters	distance	displacement
meters/sec	speed	velocity
meters/sec^2	(no scalar form of acceleration exists in physics)	acceleration

Addition

Scalars are easy to add:

2 apples + 3 apples = 5 apples

But vectors are more complicated:

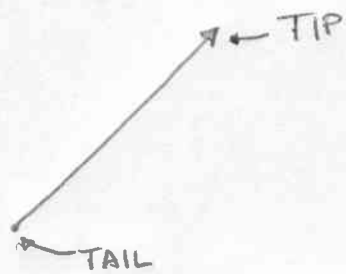
10 miles North + 2 miles North = 12 miles North

<u>but</u>

10 miles North + 2 miles East ≠ 12 miles !!!

To add vectors, you add "tip to tail":

- A vector is represented on paper by an arrow



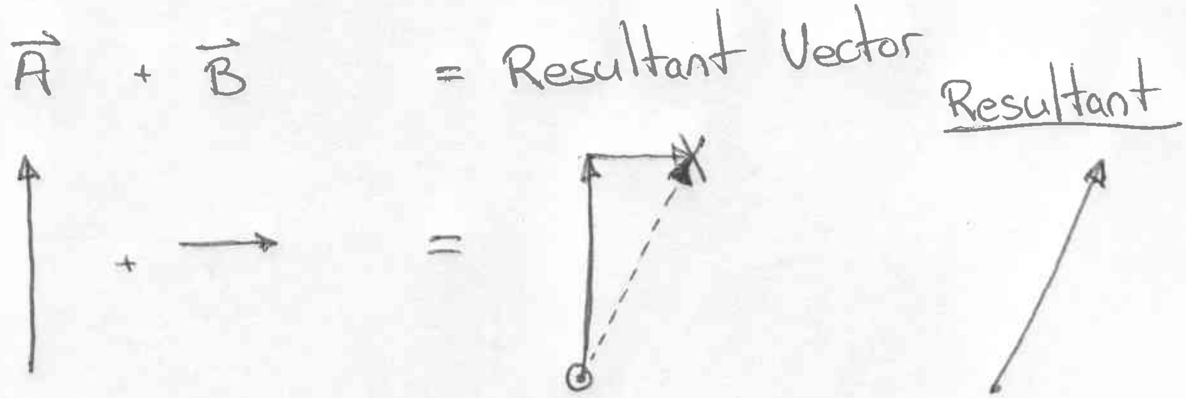
<u>Length</u> shows magnitude
<u>Direction</u> shows direction

- For vectors $\vec{A}$ and $\vec{B}$, add as follows to find the resultant: $\vec{A} + \vec{B}$

1. Pick a starting point. Label it with a circle "o"
2. Starting at the "o", draw the first vector with the tip pointing away.
3. Starting at the tip of the first vector, draw the second vector so that the tail of the second connects to the tip of the first.

Addition (continued)

4. Draw an "x" at the tip of the final arrow.
5. Draw a dashed line from the ~~tail~~ "o" to the "x". This is your resultant vector.



Graphical Addition: If you use a ruler and a protractor, and if you measure precisely, you can find the resultant of any number of vectors. However, accuracy of your solution will depend on your skill with the ruler and protractor.

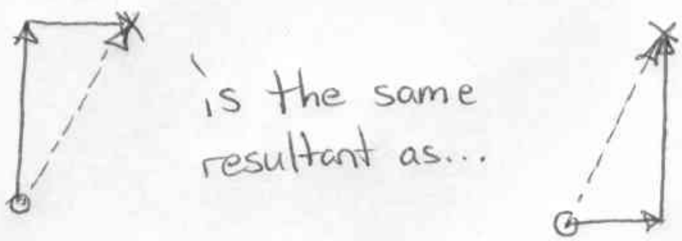
For AP, the graphical method isn't accurate enough. You will need the . . . ,

Component Method of Vector Addition:

... Before we can begin using this hyper-accurate method of vector addition, we must realize a few things:

1. The commutative property, $\vec{A}+\vec{B}=\vec{B}+\vec{A}$, holds for vector addition:

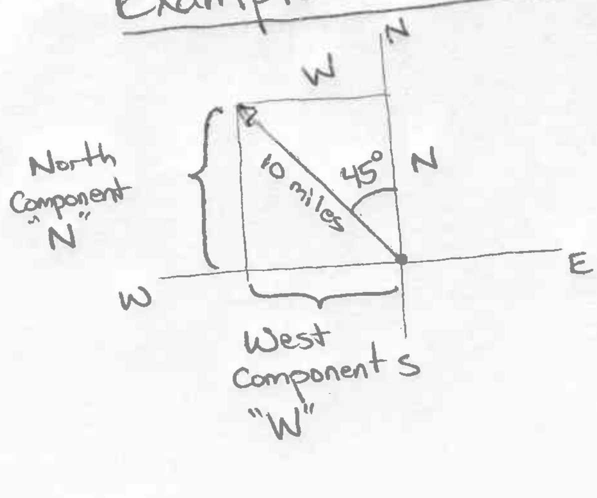
$\vec{A}+\vec{B}$



→ It doesn't matter which order you add the vectors, as long as you follow the "tip to tail" rule correctly.

2. Any vector can be broken down into its components.

Example: 10 miles North-West



consists of a North component:

$$\cos 45° = \frac{N}{10 \text{ miles}}$$

$$N = (10 \text{ miles}) \cos 45°$$

$$N = 7.07 \text{ miles North}$$

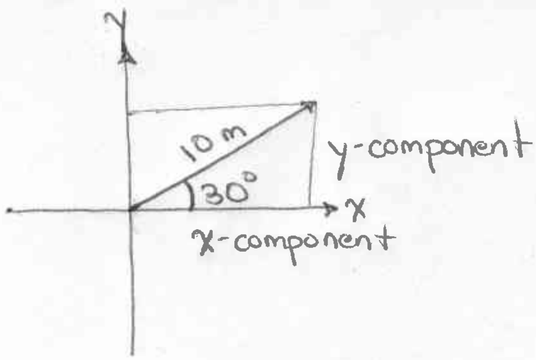
and a West component:

$$\sin 45° = \frac{W}{10 \text{ miles}}$$

$$W = 7.07 \text{ miles West}$$

Component Method (continued):

Example: 10 m @ 30° (up from the x-axis)



x-component:

$$\cos 30^\circ = \frac{x}{10\text{ m}}$$

$$x = (10\text{ m})\cos 30^\circ$$

$x = 8.66\text{ m}$ in the x-direction

y-component:

$$\sin 30^\circ = \frac{y}{10\text{ m}}$$

$y = 5\text{ m}$ in the y-direction

3. Lastly, you must realize that if you add all the individual x components for all the vectors, it will tell you exactly how far you will go in the x direction. Likewise, if you add all the y components, you know how far you will have gone in the y direction.

In the end, the resultant vector is what you get when you stick the sum of the x vectors together with the sum of the y vectors.

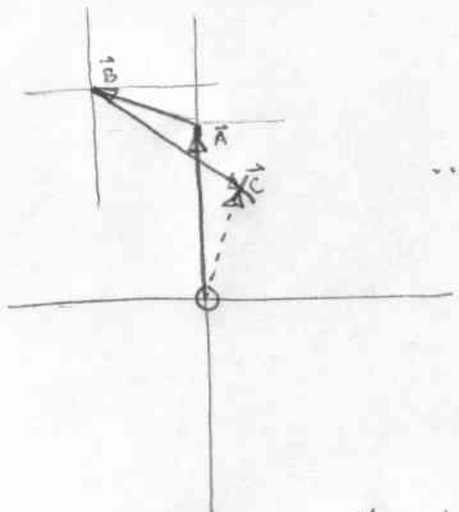
Really heinous example:

What do you get when you add:

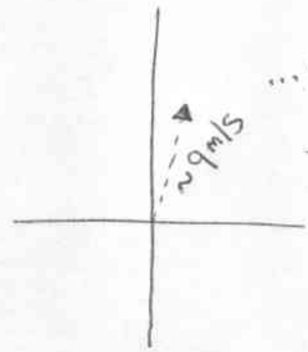
$\vec{A}$: 12 m/s @ +90° from x axis

$\vec{B}$: 7 m/s @ +150° from x axis

$\vec{C}$: 10 m/s @ −40° from x axis



...graphically, resultant is:

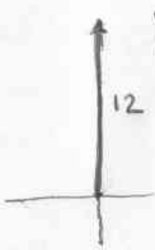


...or about 9 m/s somewhere just to the right of the ~~x~~ y axis... (remember this for later)

...quantitatively, we split vectors into x and y components:

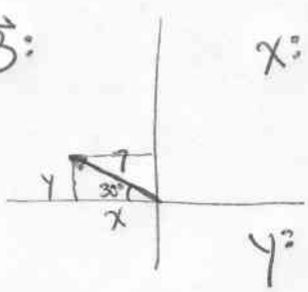
	x	y
$\vec{A}$	0	+12
$\vec{B}$	−6.1	+3.5
$\vec{C}$	+7.7	−6.4
resultant	+1.6	+9.1

$\vec{A}$:



x: 0 m/s
y: +12 m/s

$\vec{B}$:



x: $\cos 30° = \frac{x}{7}$

$x = 7 \cos 30°$

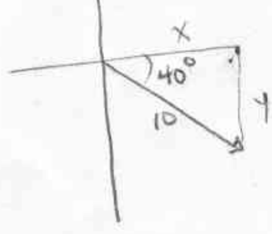
$\boxed{x = {}^{-}6.1}$

y: $\sin 30° = \frac{y}{7}$

$y = 7 \sin 30°$

$\boxed{y = {}^{+}3.5}$

$\vec{C}$:



x: $\cos 40° = \frac{x}{10}$

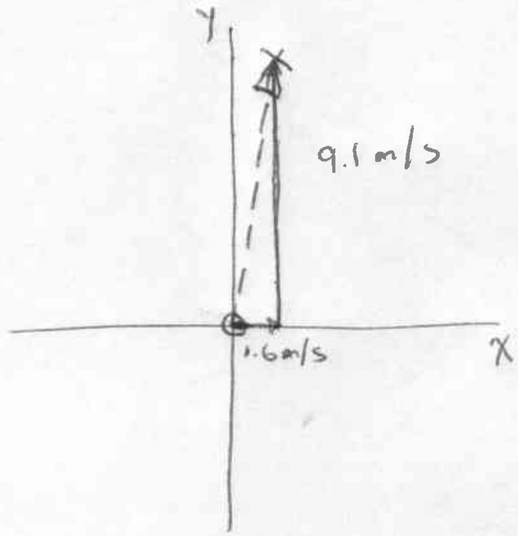
$\boxed{x = 7.7}$

y: $\sin 40° = \frac{y}{10}$

$\boxed{y = {}^{-}6.4}$

Solution to Really Heinous Example

So when we added $\vec{A}$, $\vec{B}$, and $\vec{C}$, we found a resultant which has an x-component of +1.6 m/s, and a y-component of +9.1 m/s. But we still need to put them back together:



We can find the magnitude with the Pythagorean Theorem:

$$(\text{magnitude})^2 = 1.6^2 + 9.1^2$$

$$\text{magnitude} = 9.2 \text{ m/s}$$

We can find the direction with trigonometry:

$$\tan\theta = \frac{9.1}{1.6}$$

$$\theta = 80^\circ$$

So our final answer is:

$$\boxed{\vec{A} + \vec{B} + \vec{C} = 9.2 \text{ m/s } @ {}^{+}80^\circ \text{ from the x-axis}}$$

↳ If you check this against the graphical method, you will find they both agree.

Subtraction

To subtract vectors, you simply add the <u>negative</u> of the vector, meaning the vector with the same magnitude, but opposite direction.

<u>for example...</u>

$\vec{A}$ 

$\vec{B}$ 

$\vec{A} + \vec{B}$:  resultant: 

$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$:  resultant: 

$\vec{B} - \vec{A} = \vec{B} + (-\vec{A})$:  resultant: 

$\therefore\ \vec{A} - \vec{B} \neq \vec{B} - \vec{A}$, just like $2 - 3 \neq 3 - 2$

Multiplication and Division

Multiplying and dividing two scalars are easy:

$$2 \times 3 = 6$$

$$\frac{4}{2} = 2$$

Multiplying or dividing a scalar times a vector or a vector by a scalar is also easy:

$$2(10 \text{ miles North}) = 20 \text{ miles North}$$

$$\frac{10 \text{ miles North}}{5} = 2 \text{ miles North}$$

Multiplying two vectors is the subject of an advanced math course (multivariable calculus with vector analysis), so we won't discuss it here.

... Thats it for this edition of "Fun with Vectors". Happy Homeworking!