Momentum - Cons of Momentum and Imp-Mom Thm

Note Title 10/15/2010

Review

Definition of momentum ($\vec{P}$)

- $\vec{P} = m\vec{V}$
- Units $\doteq [\frac{kg \cdot m}{s}]$
- $\vec{P}$ is a vector quantity in the direction of $\vec{v}$

Definition of Impulse ($\vec{J}$)

- $\vec{J} = \int \vec{F} dt$
- Units $\doteq [N \cdot s]$

Impulse-Momentum Theorem

- An impulse imparted on an object changes the momentum of the object
- $\vec{J} = \Delta \vec{P}$
- $[Ns] \doteq [\frac{kg \cdot m}{s}]$
- $\int \vec{F} dt = \Delta (m\vec{V})$
- For constant force and constant mass...

$$\vec{F} \Delta t = m \Delta \vec{V}$$

Law of Conservation of Momentum

- $\Sigma P_0 = \Sigma P_1 = \Sigma P_2 = \ldots = \text{constant}$
- For a system of 2 objects...

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

The Real Newton's 2nd Law

As translated by Andrew Motte...

"The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed."

Mathematically:

"alteration of motion" = rate of change of momentum

$$= \frac{d\vec{p}}{dt}$$

"motive force impressed" = $\vec{F}$

Therefore, Newton's 2nd Law is most accurately expressed as:

$$\boxed{\vec{F} = \frac{d\vec{p}}{dt}}$$

This is much more useful than the simplification you used last year ($\vec{F} = m\vec{a}$). It allows us to solve problems where mass is not constant!

$$\vec{F} = \frac{d(mv)}{dt}$$

constant velocity ↙ $\vec{F} = \vec{v}\frac{dm}{dt}$

constant mass ↘ $\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}$

The Impulse-Momentum Theorem is merely a restatement of Newton's 2nd Law for constant mass:

$$\int \vec{F} dt = m \Delta \vec{v}$$

$$\downarrow$$

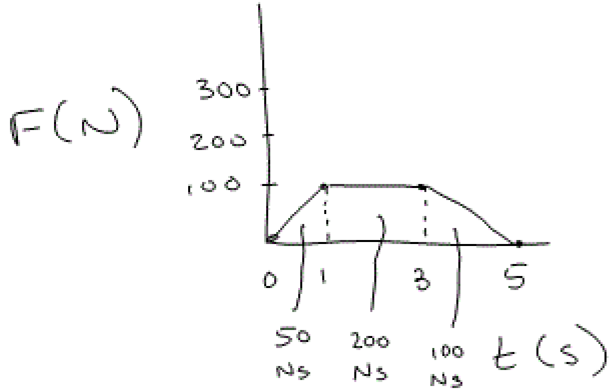
$$\vec{F} dt = m d\vec{v}$$

$$\downarrow$$

$$\boxed{F = m \frac{dv}{dt}}$$

Graphing with N2L / Impulse-Momentum Theorem

Since $\Delta \vec{p} = \int \vec{F} dt$, we can find the change in momentum of an object from a graph of F vs t:

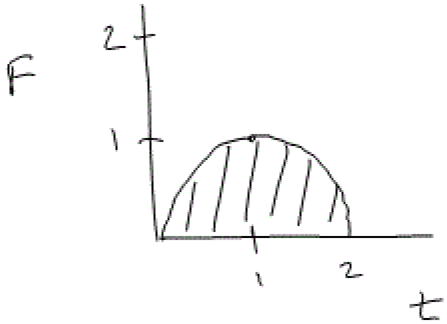


total impulse = 350 NS

$\hookrightarrow \Delta p = 350 \frac{kg\,m}{s}$

Or, if force is given by a more complex function:

$F = -t^2 + 2t$



$$\text{Impulse} = \Delta P = \int F dt$$

$$= \int_0^2 (-t^2 + 2t) dt$$

$$= -\frac{t^3}{3} + t^2 \Big|_0^2$$

$$= -\frac{8}{3} + 4$$

$$\boxed{= \frac{4}{3}}$$