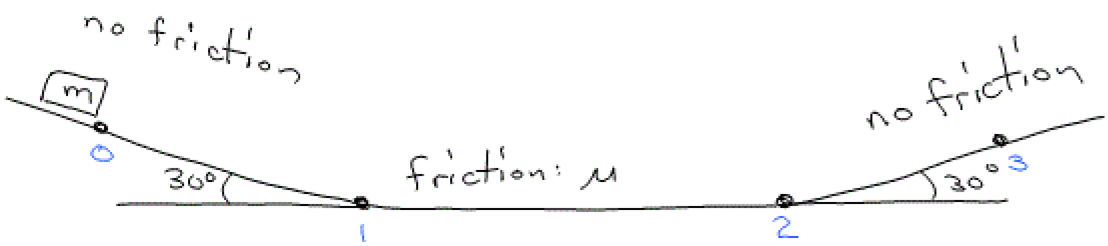
Energy - State Variables, Deriving Potential Energy

Note Title 10/11/2010

State Variables

One very useful technique when tackling conservation of energy problems (think rollercoasters) is to label each point of interest on the diagram with a letter or number. For example:



Points 0, 1, 2, 3 are "states". The status of the block at each state is given by a set of "state variables":

h_0	h_1	h_2	h_3
v_0	v_1	v_2	v_3
U_0	U_1	U_2	U_3
K_0	K_1	K_2	K_3

If we were asked how high the block had to be lifted at state 0 (h_0) if it had a velocity of v_2 at state 2, we can use the law of Conservation of Energy, and we can neglect state 1:

$$\underline{\text{Total Energy @ }0} = \underline{\text{Total Energy @ }2 + \text{Friction Work }1\to2}\ ^{**}$$

$$mgh_0 = \tfrac{1}{2}mv_2^2 + \mu mgd_{1-2}$$

**Note that since work is <u>NOT</u> a state variable, you must account for it along the way. Since some of the total energy you started with at state 0 was converted to friction work by state 2, the friction work goes on the state 2 side.

Derivation of Potential Energy Types

- All the potential energy relationships we have learned so far ($U_g = mgh$, $U_k = \frac{1}{2}kx^2$, etc.) require two assumptions:
  1. A given equation for force: Gravity $F_g = mg = \text{constant}$
     spring $F_k = -kx \propto x$
  2. The force is conservative: $\Delta U_{Force} = -W_{Force} = -\int \vec{F} \cdot d\vec{r}$

- The derivations:

Easy

Gravity $U_g = mgh$

$F_g = mg$

$\Delta U_g = -W_g$

If $U_0 = 0$ and up ↑ is positive:

$U_g - \cancel{U_{0g}}^{0} = -\int F_g \, dh$

$U_g = -\int_0^h (-mg)\, dh$

$U_g = +mg \int_0^h dh$

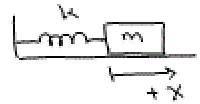
$U_g = mg(h)\Big|_0^h$

$U_g = mgh - mg(0)$

$\boxed{U_g = mgh}$

Medium

Linear Spring $U_k = \frac{1}{2}kx^2$



$F_k = -kx$

$\Delta U_k = -W_k$

$U_k - \cancel{U_{0k}}^{0} = -\int F_k \, dx$

$U_k = -\int_0^x (-kx)\, dx$

$U_k = k\int_0^x x\, dx$

$U_k = k\left(\frac{1}{2}x^2\right)\Big|_0^x$

$U_k = \frac{1}{2}kx^2 - \frac{1}{2}k(0)^2$

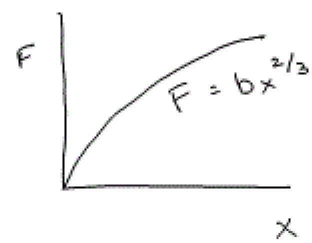
$\boxed{U_k = \frac{1}{2}kx^2}$

Hard

Nonlinear spring

Example: The bow we used in class

$U = ???$ (depends)



$\Delta U = -W_k$

$U = -\int_0^x F\, dx$

$U = -\int_0^x (-bx^{2/3})\, dx$

$U = b\int_0^x x^{2/3}\, dx$

$U = b\left(\frac{3}{5}x^{5/3}\right)\Big|_0^x$

$\boxed{U = \frac{3b}{5}x^{5/3}}$