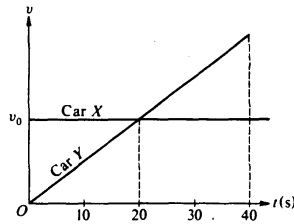


MULTIPLE CHOICE

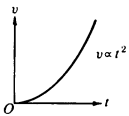


At time $t = 0$, car X traveling with speed v_0 passes car Y, which is just starting to move. Both cars then travel on two parallel lanes of the same straight road. The graphs of speed v versus time t for both cars are shown above.

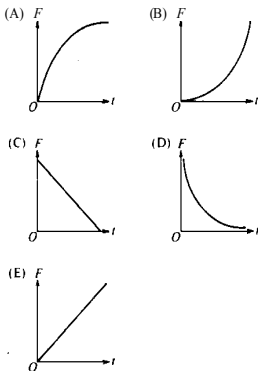
- Which of the following is true at time $t = 20$ seconds?

(A) Car Y is behind car X. (B) Car Y is passing car X. (C) Car Y is in front of car X.
(D) Both cars have the same acceleration. (E) Car X is accelerating faster than car Y.
- From time $t = 0$ to time $t = 40$ seconds, the areas under both curves are equal. Therefore, which of the following is true at time $t = 40$ seconds?

(A) Car Y is behind car X. (B) Car Y is passing car X. (C) Car Y is in front of car X.
(D) Both cars have the same acceleration. (E) Car X is accelerating faster than car Y.



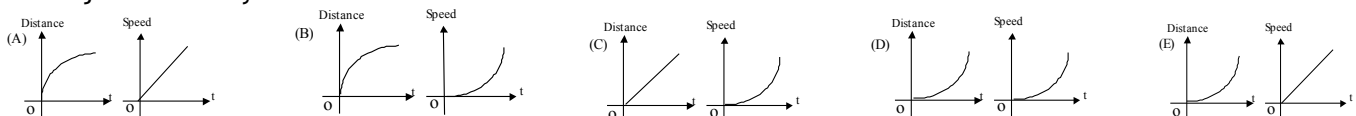
- The parabola above is a graph of speed v as a function of time t for an object. Which of the following graphs best represents the magnitude F of the net force exerted on the object as a function of time t ?



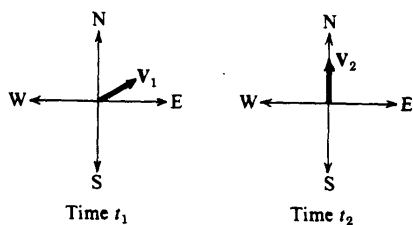
- A body moving in the positive x direction passes the origin at time $t = 0$. Between $t = 0$ and $t = 1$ second, the body has a constant speed of 24 meters per second. At $t = 1$ second, the body is given a constant acceleration of 6 meters per second squared in the negative x direction. The position x of the body at $t = 11$ seconds is

(A) +99 m (B) +36 m (C) -36 m (D) -75 m (E) -99 m

- Which of the following pairs of graphs shows the distance traveled versus time and the speed versus time for an object uniformly accelerated from rest?



6. An object released from rest at time $t = 0$ slides down a frictionless incline a distance of 1 meter during the first second. The distance traveled by the object during the time interval from $t = 1$ second to $t = 2$ seconds is
(A) 1 m (B) 2 m (C) 3 m (D) 4m (E) 5 m



7. Vectors V_1 , and V_2 shown above have equal magnitudes. The vectors represent the velocities of an object at times t_1 , and t_2 , respectively. The average acceleration of the object between time t_1 and t_2 was
(A) zero (B) directed north (C) directed west (D) directed north of east (E) directed north of west
8. A projectile is fired from the surface of the Earth with a speed of 200 meters per second at an angle of 30° above the horizontal. If the ground is level, what is the maximum height reached by the projectile?
(A) 5 m (B) 10 m (C) 500 m (D) 1,000 m (E) 2,000 m
9. A particle moves along the x-axis with a nonconstant acceleration described by $a = 12t$, where a is in meters per second squared and t is in seconds. If the particle starts from rest so that its speed v and position x are zero when $t = 0$, where is it located when $t = 2$ seconds?
(A) $x = 12$ m (B) $x = 16$ m (C) $x = 24$ m (D) $x = 32$ m (E) $x = 48$ m

Questions 10-11

An object moving in a straight line has a velocity v in meters per second that varies with time t in seconds according to the following function.

$$v = 4 + 0.5 t^2$$

10. The instantaneous acceleration of the object at $t = 2$ seconds is
(A) 2 m/s^2 (B) 4 m/s^2 (C) 5 m/s^2 (D) 6 m/s^2 (E) 8 m/s^2
11. The displacement of the object between $t = 0$ and $t = 6$ seconds is
(A) 22 m (B) 28 m (C) 40 m (D) 42 m (E) 60 m
12. A rock is dropped from the top of a 45-meter tower, and at the same time a ball is thrown from the top of the tower in a horizontal direction. Air resistance is negligible. The ball and the rock hit the level ground a distance of 30 meters apart. The horizontal velocity of the ball thrown was most nearly
(A) 5 m/s (B) 10 m/s (C) 14.1 m/s (D) 20 m/s (E) 28.3 m/s
13. Two people are in a boat that is capable of a maximum speed of 5 kilometers per hour in still water, and wish to cross a river 1 kilometer wide to a point directly across from their starting point. If the speed of the water in the river is 5 kilometers per hour, how much time is required for the crossing?
(A) 0.05 hr (B) 0.1 hr (C) 1 hr (D) 10 hr
(E) The point directly across from the starting point cannot be reached under these conditions.

FREE RESPONSE

1. Given the following equation for velocity as a function of time:

$$v(t) = 3t^2 + 2t + 4$$

- a. Determine acceleration as a function of time $a(t)$.
- b. Assuming the vehicle started out at a position of $x=6$, determine position as a function of time $x(t)$.

2. Given the following equation for velocity as a function of time:

$$v = \frac{dx}{dt} = -4 \sin 3t$$

- a. Determine acceleration as a function of time $a(t)$.
- b. Assuming the cart started out with a position $x(0) = 3$, determine position as a function of time $x(t)$.

3. Given the following equation for charge q as a function of time:

$$q = Q_0 \left(1 - e^{-\frac{t}{RC}} \right)$$

- a. Determine current I as a function of time $I(t) = dq/dt$.

4. Find dy/dx of the following equation: $y = \ln(x^2 + 2x)$

5. Given: $-kx^3 = -\frac{dU}{dx}$ Find $U(x)$ when $U(0) = 0$.

6. Given: $\frac{dN}{dt} = -\lambda N$ Find $N(t)$ when $N(0) = N_0$ and $N(t) = N$.

1. $v(t) = 3t^2 + 2t + 4$ $v = \frac{dx}{dt}$ $a = \frac{dv}{dt}$

a) $a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 + 2t + 4) = 6t + 2$

b) $x(t) = \int v dt = \int (3t^2 + 2t + 4) dt$
 $= \frac{3t^3}{3} + \frac{2t^2}{2} + 4t + C = t^3 + t^2 + 4t + C$

$0 = 1 + 1 + 4 + C \Rightarrow C = 6$

$x(t) = t^3 + t^2 + 4t + 6$

8/30/2010

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2. $v = \frac{dx}{dt} = -4 \sin 3t$

a) $\frac{dv}{dt} = -4 \sin 3t(3) = -12 \cos 3t$

b) $x(t) = \int v dt = -\int 4 \sin 3t$
 $= -\frac{4}{3}(-\cos 3t) = \frac{4}{3} \cos 3t + C$ $3 = \frac{4}{3} + C \Rightarrow$

$\therefore C = \frac{9}{3} - \frac{4}{3} = \frac{5}{3}$ $x(t) = \frac{4}{3} \cos 3t + \frac{5}{3}$

9/2/2009

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3. Given $\rightarrow q = Q_0 \left(1 - e^{-\frac{t}{RC}}\right)$

$q = Q_0 - Q_0 e^{-\frac{t}{RC}}$

$i = \frac{dq}{dt} = 0 + \frac{Q_0}{RC} e^{-\frac{t}{RC}}$

9/2/2009

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4. Given $\rightarrow y = \ln(x^2 + 2x)$

$\Rightarrow u = x^2 + 2x$

$\Rightarrow du = (2x + 2) dx$

$\therefore \ln(x^2 + 2x) = \ln u$

$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = \frac{1}{u} \frac{du}{dx} = \frac{1}{x^2 + 2x} \frac{2x + 2}{1}$

$\frac{dy}{dx} = \frac{2x + 2}{x^2 + 2x}$

9/2/2009

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5. Given $\rightarrow F = -kx^3 = -\frac{dU}{dx}$

$\frac{dU}{dx} = kx^3 \Rightarrow dU = kx^3 dx$

$\Rightarrow \int dU = \int kx^3 dx$

$\Rightarrow U = \frac{k}{4} x^4 + C$ $0 = 0 + C$

$U(x) = \frac{k}{4} x^4$

9/2/2009

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6. Given $\rightarrow \frac{dN}{dt} = -\lambda N$

$\Rightarrow dN = -\lambda N dt \Rightarrow \frac{dN}{N} = -\lambda dt$

$\Rightarrow \int_{N_0}^N \frac{dN}{N} = -\lambda \int_0^t dt$ $\ln N - \ln N_0 = -\lambda t$

$\Rightarrow \ln \frac{N}{N_0} = -\lambda t$ anti-logs $\Rightarrow \frac{N}{N_0} = e^{-\lambda t}$

$N = N_0 e^{-\lambda t}$

9/2/2009

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