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Upon completion of this unit, you should understand:

1. How parallax and binocular vision are used as aides in locating objects.

2. The principles of reflection and refraction so you can:
   a. Determine how the speed of light changes when light passes from one medium into another.
   b. Show on a diagram the direction of reflected and refracted rays.
   c. Use Snell’s Law to relate the directions of the incident ray and the refracted ray, and the indices of refraction of the media.
   d. Identify conditions under which total internal reflection will occur.
   e. Explain the formation of mirages.
   f. Describe the dispersion of white light by a prism.

3. Image formation by plane or spherical mirrors so you can:
   a. Relate the focal point of a spherical mirror to its center of curvature.
   b. Given a diagram of a mirror (with the focal point for a curved mirror shown), locate by ray tracing the image of a real object and determine whether the image is real or virtual, upright or inverted, enlarged or reduced in size.

4. Image formation by converging or diverging lenses so you can:
   a. Determine whether the focal length of a lens is increased or decreased as a result of a change in the curvature of its surfaces or its surfaces or in the index of refraction of the material of which the lens is made or the medium in which it is immersed.
   b. Determine by ray tracing the location of the image of a real object located inside or outside the focal point of the lens, and state whether the resulting image is upright or inverted, real, or virtual.
   c. Use the thin lens equation to relate the object distance, image distance, and focal length for a lens, and determine the image size in terms of the object size.
   d. Analyze simple situations in which the image formed by one lens serves as the object for another lens.
## Mirror and Lens Equations

- \( s_o \) – object distance
- \( s_i \) – image distance
- \( f \) – focal point
- \( R \) – center of curvature
- \( M \) – magnification ratio

\[
f = \frac{R}{2}
\]

\[
\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}
\]

\[
M = \frac{h_{im}}{h_{ob}} = -\frac{s_i}{s_o}
\]

### Sign Conventions for Mirrors

| \( s_o \) | + | object is in front of mirror (real object) |
| \( s_o \) | - | object is in back or mirror (virtual object) |
| \( s_i \) | + | image is in front of mirror (real image) |
| \( s_i \) | - | image is in back of mirror (virtual image) |
| \( f \) | + | focal point is in front of the mirror (concave) |
| \( f \) | - | focal point is in back of the mirror (convex) |
| \( M \) | + | image is erect |
| \( M \) | - | image is inverted |

### Sign Conventions for Lenses

| \( s_o \) | + | object is in front of lens (real object) |
| \( s_o \) | - | object is in back or lens (virtual object) |
| \( s_i \) | + | image is in back of lens (real image) |
| \( s_i \) | - | image is in front of lens (virtual image) |
| \( f \) | + | converging lens |
| \( f \) | - | diverging lens |
| \( M \) | + | image is erect |
| \( M \) | - | image is inverted |
Mirror Diagrams

Make ray diagrams to show images in the following and then complete the chart.
Mirror Summary Chart

Use the sheet of diagrams to complete the chart below.

I. Write in the image position. (Use letters A - D, F, or 2F.)

II. Indicate the type of image. (Real or Virtual.)

III. Indicate the orientation of the image. (Upright or Inverted.)

IV. Indicate the size of the image $H_1$ compared to the object size $H_0$. (Smaller, Equal, or Larger)

![Diagram of a concave mirror with object positions labeled A, B, C, D, and image positions 2F, F, between the mirror and a line representing the object position]

<table>
<thead>
<tr>
<th>Object Position</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Image Position</td>
<td>Real Image</td>
<td>Virtual Image</td>
<td>Upright</td>
</tr>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2F</td>
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<td>B</td>
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<td>D</td>
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</tbody>
</table>
Lens Diagrams

Make ray diagrams to show images in the following and then complete the chart.
Lens Summary Chart

Use the sheet of diagrams to complete the chart below.

I. Write in the image position. (Use letters A - M, F, or 2F.)

II. Indicate the type of image. (Real or Virtual.)

III. Indicate the orientation of the image. (Upright or Inverted.)

IV. Indicate the size of the image $H_i$ compared to the object size $H_o$. (Smaller, Equal, or Larger)

![Diagram of lens with labels A to M]

<table>
<thead>
<tr>
<th>Object Position</th>
<th>Image Position</th>
<th>Real Image</th>
<th>Virtual Image</th>
<th>Upright</th>
<th>Inverted</th>
<th>$H_i &lt; H_o$</th>
<th>$H_i = H_o$</th>
<th>$H_i &gt; H_o$</th>
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<tbody>
<tr>
<td>A</td>
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<td>2F</td>
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Reflection From A Plane Mirror

A. Place side A of the work sheet face up on a sheet of cardboard. Support mirror as per example at center table. The silvered surface (back edge) of the front mirror must rest on the line. Place a florist pin at the point marked 0. Place a straight pin at each of the points marked 1. Look along the surface of the paper. The two straight pins should lie on the same straight line as the image of the florist pin. Leave the mirror and florist pin in place. Remove the two straight pins. Look at the image of the florist pin from the right side of the mirror. Line up the two straight pins with the image of the florist pin. Label these points 2. Repeat this from a third angle. You will analyze this later.

B. You will need two objects: a pencil and a pen, two thumbs. Close one eye. Hold the pencil at arm's length. Hold the pen a foot closer to you so it lines up with the pencil. Move your head to the right. Think of the pen as being fixed in place. The pencil seems to move to the right. Move the pencil so it is now a foot closer to you than the pen. Move your head to the right. If you think of the pen as being fixed in place, which way does the pencil move. This apparent motion is an example of parallax. Where must the pencil be placed to eliminate parallax?

Set up the two mirrors on the lines on side B of the work sheet. Place one florist pin at point 0. Place a second florist pin at point 1. As you move your head does the second pin appear to be closer or farther than the image? Move the second pin to point 2. Is it too close or too far? Move the second pin to point 3. Is there any parallax now? Where is the image? Use parallax to find the other two images. Label each image point.

Analysis:

A. Draw the path the light appears to be traveling on from the image to the eye as a dotted line. Use a new eye for each sighting. Label the position of the image. Use a solid line to draw the path the light actually takes in going from the object to the mirror to the eye. At the point this path touches the mirror's reflecting surface, draw a normal (see p. 78 of text). Measure \( \theta_i \) and \( \theta_r \), and record these. Measure and record the distances the object (D₀) and image (D₁) are from the mirror. Answer the questions.

B. The path that the light actually travels on and the path that it appears to follow in forming the 1st image are already drawn on the sheet. Draw the paths taken to form the other two images. Use the same eye as the 1st. Be mindful of the conclusions from part A. Answer the questions.
Cougars Love Physics!
### Questions:

1. What relationship exists between the angles of incidence and reflection in each of the trials you did?

2. What relationship exists between \( d_o \) and \( d_i \)?

3. For trial #1, measure the length of the actual light ray, from the object to the mirror to the eye. Compare this to a measurement of the length of the apparent light ray, from the image to the eye.

\[ d_i = \underline{\hphantom{00000}} \]

\[ d_o = \underline{\hphantom{00000}} \]
Questions:

1. If the pin behind the mirror is further from the mirror than the image and you move your head to the right, the pin behind the mirror will move which direction?

2. If the pin behind the mirror is closer to the mirror than the image is and you move your head to the left, the pin behind the mirror will move which direction?

3. How does the distance from the corner image directly to the eye compare to the distance from the object pin to the mirror to the eye?
Curved Mirror Lab

In this lab you will become familiar with the images formed by a curved mirror. You will find where an object must be placed to form real and virtual images (see page 732); when the images will be upright or inverted; and when the images are magnified or reduced in size. You will also find the relationship between the distance from the object to the mirror \(D_o\) and the distance from the image to the mirror \(D_i\). You will find this relationship by collecting and graphing appropriate data. Graph \(D_i\) vs \(D_o\).

You will also graph \(1/D_i\) vs \(1/D_o\).

You will turn in data tables, the graph (graded on the accuracy of the data and clarity of presentation), and answers to questions that follow. Some of the questions can be answered from observations made in the lab, the graphs will be needed for others, and you may need to refer to your textbook for some. Your sheet of diagrams will also be useful.

Finding real images

Tape down a piece of white paper ticker tape about 1 1/4 m (4 feet) long. Support the mirror at one end of the tape with the concave side facing the far end of the tape. Mark the position of the focus and of the front surface of the mirror on the paper tape. The focus is 10.6 cm from the front surface of the mirror. Do this by first outlining the block supporting the mirror. Insert a plastic ruler into the saw cut in the block so the zero mark is touching the front surface of the mirror. Make a mark on the paper tape at the 10.6 cm point of the ruler. This is the focus of the mirror. Keep the ruler in the same place and remove the wooden block and mirror. Make another mark on the paper tape 10.6 cm back from the focus. This is where the front of the mirror had been. Replace the wooden block and mirror where it had been.

The object you will use here is a small light bulb. Plug the light bulb in and place it at the other end of the tape. Use an index card as a screen for the light and look for the image near the focus. Bring it into sharp focus. Mark the position of the filament of the bulb as O-1 and the position of the index card as I-1. Repeat this with the bulb in different positions to find a total of ten ordered pairs of object and image positions: five with the bulb between the end of the tape and a point 20 cm from the mirror, and five more with the bulb between the point 20 cm from the mirror and a point 12 cm from the mirror.

As well as shining the image onto the card you need to view the image by looking into the mirror from the end of the table.

After the ten pairs of points are marked on the paper tape, remove the block and mirror. Measure the distances from the object and image points to the point where the front surface of the mirror had been. Measure to the nearest tenth of a cm.
Viewing Virtual Images

Now position the mirror so you can look at it from both sides. Place an object like a pen or pencil close to the concave side of the mirror and look at the image in the mirror. Is the light that forms the image passing through the spot where you see the image? Is the image upright or inverted? Slowly move the object away from the mirror. What happens to the size of the image and its location?

Place an object like a pen or pencil close to the convex side of the mirror and look at the image in the mirror. Is the light that forms the image passing through the spot where you see the image? Is the image upright or inverted? Slowly move the object away from the mirror. What happens to the size of the image and its location?
1. If the object starts very far from the mirror and then moves toward the mirror, the size of the image will
   a. increase
   b. decrease
   c. stay the same
   and the image will
   d. move closer to the mirror.
   e. move farther away from the mirror.
   f. stay the same distance from the mirror.

2. The focus is one reference point on the axis of a curved mirror. Another is the center of curvature. What is the relationship between their distances from the mirror? (See diagram below.)

3. When the object is located at the center of curvature, the image is
   a. farther from the mirror than the object
   b. closer to the mirror than the object
   c. the same distance from the mirror as the object
   and it is
   d. larger than the object
   e. smaller than the object
   f. the same size as the object
   and the image is
   g. upright
   h. inverted

4. If the object starts at the center of curvature and is moved toward the mirror, the size of the image will
   a. increase
   b. decrease
   c. stay the same
   and the image will
   d. move closer to the mirror.
   e. move farther away from the mirror.
   f. stay the same distance from the mirror.

5. If the object starts at the center of curvature and is moved away from the mirror, the size of the image will
   a. increase
   b. decrease
   c. stay the same
   and the image will
   d. move closer to the mirror.
   e. move farther away from the mirror.
   f. stay the same distance from the mirror.

6. What are the two locations where the image and object are in the same position and are the same size?
7. If the object starts next to the concave mirror and is moved toward the focus, the size of the image will
   a. increase
   b. decrease
   c. stay the same
   and the image will
   d. move closer to the mirror.
   e. move farther away from the mirror.
   f. stay the same distance from the mirror.

8. If the object starts next to the convex mirror and is moved away from the mirror, the size of the image will
   a. increase
   b. decrease
   c. stay the same
   and the image will
   d. move closer to the mirror.
   e. move farther away from the mirror.
   f. stay the same distance from the mirror.

9. If the object is more than 20 cm from the concave mirror, the image is
   a. real
   b. virtual
   and the image is
   c. upright
   d. inverted

10. If the object is less than 10 cm from the mirror, the image is
    a. real
    b. virtual
    and the image is
    c. upright
    d. inverted

11. At what position of the object does the type of image and its orientation change?

12. Calculate the slope of the second graph; include units.

13. What is the relationship between the Y-intercept and the focal length of the mirror? If you cannot tell from your data, pool yours with a few other groups and get an average.

14. What is the equation relating $D_o, D_i$ and $f$? Use your second graph and your answer to the preceding question. (No numbers should appear in this equation.)
13 Vibrations and Waves

Periodic Motion and Distortions of the Medium

Motion that repeats itself over and over is very common and applies to rotation, to oscillating springs, vibrating reeds, to waves and many others. Whenever there is a restoring force back to equilibrium for a displacement we get oscillatory motion of one type or another. The oscillations can be simple (without change from cycle to cycle), with diminished amplitudes (damping forces are present), or increased amplitudes (resonance phenomenon in forced oscillations), or combinations and variations of the above such as coupled oscillations, etc.

13.1 Hooke's Law

Many forces encountered in everyday living are elastic in nature as long as the elastic limit has not been exceeded, for example, the slight bending of a rod or plank (like a diving board), the stretch of a spring or even a straight wire, the bounce in a tire full of air, and etc. For all these situations, the force causing the stretch or bend is proportional to the displacement. The reaction force is a restoring force trying to return everything back to equilibrium. The restoring force is then,

\[ F_x = -kx \]  

which is known as Hooke's law. Simple harmonic motion (SHM) occurs with oscillating things that obey Hooke's law. The motion is simple and each cycle is like the one before. It should be noted, however, that not all repetitive motion (simple or not) is SHM.

In SHM the period, T, is the time to complete a cycle, the maximum displacement over a cycle is the amplitude, A, and the frequency, f, is the number of cycles/s. One cycle per second is called a hertz (symbol Hz).

Since \( F = ma \), then the acceleration at any time is obtained from \( ma = -kx \) and hence,

\[ a = \frac{-kx}{m} \]  

13.2 Elastic Potential Energy

The potential energy, \( PE \), in a system obeying Hooke’s law depends on the square of the displacement with the initial value defined as zero when at equilibrium. By this definition of zero \( PE \) we define it as

\[ PE_x = \frac{1}{2}kx^2 \]  

where \( k \) is the spring constant, i.e. the proportionality constant in Hooke’s law, \( F = -kx \).

This potential energy is conservative so we can include it as part of the mechanical energy in our conservation of energy equation, namely,

\[ (KE + PE_x + PE_f)_{E} = (KE + PE_x + PE_f)_{F} \]  

If nonconservative, dissipative forces are present the work done by these forces is the energy lost from the total mechanical energy between the initial and final states, i.e. \( W_{ds} = E_f - E_i \).

13.3 Velocity as a Function of Position

Using the conservation of energy equation from the previous section and concerning ourselves only with kinetic energy and elastic potential energy, we know the total energy at maximum displacement is \( \frac{1}{2}kA^2/2 \) so

\[ \frac{1}{2}kA^2 = \frac{1}{2}mv^2 = \frac{1}{2}kx^2 \]  

and solving for \( v \)

\[ v = \pm \sqrt{\frac{k}{m}(A^2 - x^2)} \]  

13.4 Comparing Simple Harmonic Motion with Circular Motion

The vertical up and down motion of a pendulum on the rim of a uniformly rotating wheel is SHM. The same is true for the horizontal back and forth motion. The two dimensional rotating motion when projected into a one dimensional plane is SHM.

PerIOD AND FREQUENCY

The period of the projection of rotation, \( T \), is the same as the period of rotation. \( T \). Thus the velocity on the rim of the wheel is \( v_0 = 2\pi/A/T \), which is the maximum velocity of the projection. Thus, \( mv_0^2/2 = kA^2/2 \) so

\[ \frac{A}{v_0} = \sqrt{\frac{m}{k}} \]  

and so \( T = 2\pi\sqrt{\frac{m}{k}} \) (13.4)

and the frequency, \( f \), is simply \( 1/T \).

13.5 Position as a Function of Time

The position of a particle as a function of time for SHM is given by

\[ x = A \cos(\omega t) \]  

where \( \omega = 2\pi f \) and the particle is at \( A \) when \( t = 0 \).

13.6 Motion of a Pendulum

The simple pendulum is an example of SHM when the amplitude of swing is small. The restoring force is really \( F_t = -mg \sin\theta \), but when \( \theta \) is small \( \sin\theta \approx \theta \), so \( F_t = -mg\theta \approx -mg\theta \), where \( s \) is the arc length and \( L \) is the length of the string. This equation is of the right form to be Hooke’s law where the constant \( k = mg/L \). By adapting the equation for period, \( T \), from section 13.4, we get

\[ T = 2\pi\sqrt{\frac{m}{(mg/L)}} = 2\pi\sqrt{\frac{L}{g}} \]  

Thus we see that the period of the pendulum does not depend upon its mass, but only the length, \( L \), of the pendulum.

13.7 Damped Oscillations

When resistive or frictional forces are present along with Hooke’s law forces, and no external energy sources restore the lost energy due to the nonconservative forces, then the amplitude of each oscillation is diminished. We call this motion damped oscillations. The amount of damping depends upon how great the resistive forces are.
22
Reflection and
Refraction of Light

Light and EM Waves at Boundaries

In the previous chapter light was presented as being a part of the electromagnetic spectrum. Long before it was understood as an EM phenomenon it had been studied and known to be dispersed into many colors. had a very high speed, would reflect and refract, would interfere like other waves, and could be polarized. We study some of these attributes in this chapter.

22.1 The Nature of Light

Light has been demonstrated to be waves. Thomas Young showed this in 1801 by interference of two coherent waves. Before being shown that it was waves, scientists believed it was made of particles. Today it is known to be made of particles called photons and it is also waves. The energy of the photon is proportional to its frequency and all photons of a given frequency have the same amount of energy.

\[ E = hf \]  (22-17)

The constant \( h \) is known as Planck’s constant and is the same \( h \) used in describing the angular momentum and spin on the atomic level. Its SI value is \( 6.63 \times 10^{-34} \) J-s.

22.2 Measurements of the Speed of Light

Early experiments to measure the speed of light failed because the speed was so high.

ROEMER’S METHOD

By timing the moments of occultation of Jupiter’s moons by the giant planet, Ole Roemer observed that the further away the earth got on its orbit from Jupiter, the later they occurred. He assumed the extra time required was due to the travel time of the light over the extra distance. He was right and was able to measure the speed of light to an accuracy of about 2/3 of its presently known value.

FIZEAU’S TECHNIQUE

Armand FIZEAU passed a beam of light through the gaps between teeth of a rapidly rotating wheel and detected the reflected beam when it again passed the wheel. At the fastest rotation rate for light to be returned, the reflected beam passed through a gap adjacent to the original gap. From the distance traveled and the speed of rotation of the wheel, the speed of light was calculated within about 3%.

22.3 Huygens’ Principle

Huygens assumed light is waves and devised ways of understanding the way these waves would move. He devised a rule for drawing the propagation of light waves known as Huygens’ principle. The rule is:

All points on a given wavefront are taken as point sources for the production of spherical secondary waves, called wavelets, that propagate outward with speeds characteristic of waves in that medium. After some time has elapsed, the new position of the wavefront is the surface tangent to the wavelets.

22.4 Reflection and Refraction

REFLECTION OF LIGHT

As parallel light rays are incident upon a flat reflective surface, they reflect so as to make the angle of reflection equal to the angle of incidence. Both angles are measured between the ray and a line drawn perpendicular to the surface. Thus, the law of reflection is

\[ \theta_i = \theta_r \]  (22-18)

REFRACTION OF LIGHT

In passing through a flat boundary and continuing on the other side into a medium where the speed is different, a light ray will change its direction according to Snell’s law given as

\[ \frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1} = \text{constant} \]  (22-19)

The path of a light ray through a refracting surface is reversible. The path taken is independent of the direction of travel of the wave.

Light usually travels more slowly in a medium where there are atoms and electrons than in a vacuum. The change in speed is caused by a phase shift when the wave interacts with particles of the medium.

22.5 The Law of Refraction

It is convenient to define the index of refraction as

\[ n = \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v} \]  (22-20)

In passing from one medium to another, the frequency remains constant. Since the speed changes, however, the wavelength must also change. The connection
between wavelength and index of refraction can be written and Snell’s law of refraction can also be written in terms of index of refraction for each side of a boundary.

\[ \lambda_1 n_1 = \lambda_2 n_2, \quad n = \frac{\lambda_0}{\lambda_n} \]

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

### 22.6 Dispersion and Prisms

Rays of different wavelengths travel at different speeds in materials, therefore the index of refraction is a function of wavelength. Accordingly, the angle of refraction will be different for each wavelength. The shorter the wavelength the more it interacts and the slower it goes. Therefore, blue light is refracted (bent) more in going into a denser medium than red light (red wavelength – twice as long). This spreading of colors is called dispersion. With a prism light can be dispersed at each face, increasing the dispersion. This prism effect is well known to produce a beautiful color spectrum of visible light. Remember, blue light is bent the most.

### 22.7 The Rainbow

The rainbow is a good example of the dispersion of light. The light enters the droplets of water, refracts upon entering and reflects internally and finally re-emerges. Because different wavelengths are refracted by different amounts the rays become dispersed and emerge at different angles. An observer sees differing colors dependent upon the direction of observation.

### 22.8 Huygens’ Principle Applied to Reflection and Refraction

Huygens’ principle can be used successfully to construct the law of reflection because the reflected wavelets travel the same distance in the same amount of time giving the angle of reflection to be the same as the angle of incidence. In refraction, however, the refracted wavelets travel more slowly and this changes the direction of the wavefront (which is perpendicular to the light rays) thus we get Snell’s law, \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \).

### 22.9 Total Internal Reflection

Since the path of light rays is reversible we can understand how total internal reflection occurs. That is, if a light ray is inside a denser medium and is incident at an angle great enough so the exit angle is 90°, there can be no such refracted ray and all of the intensity appears in the reflected ray. This angle of incidence is called the critical angle and is given by

\[ n_1 \sin \theta_1 = n_2 \sin (\pi/2) \] so \[ \sin \theta_c = \frac{n_2}{n_1} \]

Total reflection can only occur when a light ray goes from a medium with a higher index of refraction to one that is lower.

**FIBER OPTICS**

A light beam can travel from one end of a transparent fiber to the other even though the fiber bends and curves because the beam always hits the surface at an angle greater than the critical angle and is totally internally reflected. There are a few light rays that scatter at a smaller angle and, thus, get out.
23 Mirrors and Lenses

Reflection and Refraction at Shaped Boundaries

The process of image formation is extremely useful and practical as we consider telescopes, microscopes, eyeglasses, projectors, magnifiers, etc. These involve lenses which interact with spherical wavefronts coming from point sources and reshape them to come to focus at another point, and thus make images. Both curved mirrors and curved refracting surfaces can be used for this purpose. This chapter deals with some of the simple concepts involved in creating images.

23.1 Plane Mirrors

Because of the law of reflection, light from an object striking a plane mirror reflects to give an image inside the mirror that is virtual since the light rays don’t actually go or come from there. They only appear to come from there, hence the image is virtual. The image also will be erect, the magnification (image height/object height) will be 1, and it will be the same distance from the surface (but inside) as the object is outside. The image will have a perceived left-right reversal but actually there is a front-back reversal instead. A persons image of a right hand will be a reflection of his left hand, and vice-versa. The magnification for any mirror or lens system is defined as

\[ M = \frac{\text{image height}}{\text{object height}} = \frac{h'}{h} \]  

23.2 Images Formed by Spherical Mirrors

The surface of a spherical mirror has a spherical shape, at least a segment of a sphere. It does not focus precisely because a parabolic surface is needed for that, but it approximates a parabolic surface fairly well and is much easier to make.

CONCAVE MIRRORS

If the object distance from the lens is \( p \) and the image distance is \( q \), then

\[ \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \]

where \( R \) is the radius of curvature and \( f \) is the focal length of the lens. If \( p \) is positive when to the left, then positive \( q \) is also to the left. Also \( f \) is positive. If \( q \) is positive the image is real, if \( q \) is negative it is virtual.

The magnification which is the image height divided by the object height is also given by \( M = -q/p \).

23.3 Convex Mirrors and Sign Conventions

The same lens formula applies as for the concave case but now \( R \) and hence \( f \) are negative. \( p \) is positive to the left, \( q \) negative is to the right and makes a virtual image since the light rays only appear to come to a focus on the right but cannot actually come from there.

Ray Diagrams for Mirrors

Using ray drawing one can construct the image for mirrors using four rays drawn as follows.

1. Ray 1 is drawn parallel to the principal axis and is reflected back through the focal point, \( F \).
2. Ray 2 is drawn through the focal point. Thus, it is reflected back parallel to the principal axis.
3. Ray 3 is drawn through the center of curvature, \( C \), and is reflected back on itself.
4. Ray 4 is drawn through the center of curvature (at \( R \)) and is reflected back on itself.

The intersection of any two of these rays locates the image with the third and fourth rays serving as checks.

23.4 Images Formed by Refraction

For refraction through a single curved surface and going from index of refraction \( n_1 \) to index of refraction \( n_2 \), the formula is (see the text for the derivation).

\[ \frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \]

PLANE REFRACTING SURFACES

For a plane surface, \( R \) is infinity so the term on the right side of the equation above goes to zero and we have

\[ q = -(n_2/n_1)p \]

23.5 Atmospheric Refraction

There are two naturally occurring results due to refraction in the earth’s atmosphere. First we have the visible sun after it has physically set, at sundown. It can be seen because atmospheric refraction bends the rays down to us. Second, the mirage produced by hot air near the ground making it possible to see objects above the horizon as though they were below the horizon. In the most common example, we see an image of the sky as though it had reflected off “water” on a hot, dry highway.

23.6 Thin Lenses

For thin lenses we have two radii of curvature and they are close enough together so that we can ignore the thickness of the lens. We apply the single curvature formula twice and get

\[ \frac{1}{p} + \frac{1}{q} = \frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]
A converging lens will be thicker at the center than on the edges and will have a positive focal length $f$. A diverging lens will be thicker at the edges and has a negative focal length. If the rays come from the left then $p$ is positive if on the left side of the lens and $q$ is positive if on the right, otherwise they are negative. The focal length of a converging lens is positive and of a diverging lens negative.

**RAY DIAGRAMS FOR THIN LENSES**

Three rays can readily be drawn for thin lenses and any two will suffice to determine the position of the image. The three rays are:

1. The first ray is drawn parallel to the principal axis. After being refracted by the lens, this ray passes through (or appears to come from) one of the focal points.

2. The second ray is drawn through the center of the lens. This ray continues in a straight line.

3. The third ray is drawn through the focal point, $F$, and emerges from the lens parallel to the principal axis.

**COMBINATION OF THIN LENSES**

For a combination of thin lenses the thin lens formula is applied over and over. The image distance from the first lens is determined, then with that position known, its distance to the next lens is used as the new object distance and a new image is formed, etc. This process is repeated until the final image position is known.

**23.7 Lens Aberrations**

There are several imperfections in lenses preventing perfect images. Two important ones are spherical aberrations and chromatic aberrations. Spherical aberrations are a consequence of a spherical shape not being able to give a perfect focus. We could try other shapes but they won’t work either. Spherical surfaces are the most easy to make and they work quite well. Chromatic aberrations are due to dispersion, i.e., because the index of refraction is different for each of the colors (wavelengths) then the focal length is differs for each and hence so does the image position. The variations are small, but leave fringes of color around the image. Both of these aberrations can be corrected to a certain degree. In a mirror, a parabolic shaped surface totally eliminates spherical aberration for point sources and images. Reflective surfaces have no chromatic aberration since all wavelengths reflect the same.

**23.8 Concept Statements and Questions**

1. Lenses and mirrors are used to make images of objects. When the light rays from the object actually come together to form an image, the image is real, but if they only appear to have come together (because they are actually diverging), the image is virtual.

2. The optical system (mirror or lens) can form inverted images or erect images, enlarged or reduced, real or virtual images, depending on the focal length and the position of the object.

3. The rules for drawing ray diagrams for mirrors and lenses to construct images are the same except the images for the mirror are on the opposite side as though they had been reflected (which they have).

4. Combinations of lenses can be worked by taking one lens at a time. Start with the position of the object and use the image produced by the first lens as the object of the second lens and etc.

5. How can the magnification be obtained if the image and object distances from the lens (or mirror) are known?

**23.9 Hints for Solving the Problems**

**General Hints**

1. The mirror formula and thin lens formula are identical in appearance. The only difference is the interpretation of the sign of the focal length and the sign of the image distance. The mirror is in a sense a reflection of the thin lens.

2. The hardest part of these problems is keeping the signs straight. Review the rules (from the textbook) carefully. Knowing the rules makes everything simple.

3. Study the examples carefully and practice solving problems using both the math and the ray tracing technique. Do it for every possible arrangement of object and focal point (including) signs. It will all become very clear and simple and you will gain some intuition.
1. If you stand 2.0 m in front of a plane mirror, how far away would you see the image of yourself?
   a. 1.0 m
   b. 2.0 m
   c. 4.0 m
   d. 8.0 m

2. An object is placed a distance d in front of a plane mirror. The size of the image will be
   a. half as big as the size of the object.
   b. dependent on the distance d.
   c. dependent on where you are positioned when you look at the image.
   d. twice the size of the object.
   e. the same size as the object, independent of the distance d or the position of the observer.

3. Which one of the following best describes the image of a plane mirror?
   a. real, inverted, and diminished
   b. virtual, erect, and not magnified
   c. virtual, erect, and magnified
   d. virtual, erect, and smaller

4. A person's face is 30 cm in front of a concave shaving mirror. If the image is an erect image 1.5 times as large as the object, what is the mirror's focal length?
   a. 20 cm
   b. 50 cm
   c. 70 cm
   d. 90 cm

5. An object is situated between a concave mirror's surface and its focal point. The image formed in this case is
   a. real and inverted.
   b. real and erect.
   c. virtual and erect.
   d. virtual and inverted.

6. An object is 47.5 cm tall. The image is 38.6 cm tall, and 14.8 cm from the mirror. How far is the object from the mirror?
   a. 124 cm
   b. 47.6 cm
   c. 18.2 cm
   d. 12.0 cm

7. An object is 8.90 cm tall. The image is 7.80 cm tall, and 14.8 cm from a convex mirror. What is the mirror's focal length?
   a. -120 cm
   b. -105 cm
   c. -16.9 cm
   d. -13.0 cm

8. If a material has an index of refraction of 1.461, what is the speed of light through the liquid?
   a. $2.05 \times 10^8$ m/s
   b. $2.34 \times 10^8$ m/s
   c. $1.46 \times 10^8$ m/s
   d. $4.38 \times 10^8$ m/s
9. Light passes from air to water. The incoming ray is at an angle of $17.0$ $\theta$ to the normal. The index of refraction is 1.33. What is the angle in the water?
   a. $22.9$ $\theta$
   b. $22.6$ $\theta$
   c. $18.3$ $\theta$
   d. $12.7$ $\theta$

10. Light traveling at an angle into a denser medium is refracted
   a. toward the normal.
   b. away from the normal.
   c. parallel to the normal.
   d. equally.

11. Light enters air from water. The angle of refraction will be
   a. greater than or equal to the angle of incidence
   b. less than or equal to the angle of incidence
   c. equal to the angle of incidence
   d. less than the angle of incidence

12. An index of refraction less than one for a medium would imply
   a. that the speed of light in the medium is the same as the speed of light in air.
   b. that the speed of light in the medium is greater than the speed of light in air.
   c. refraction is not possible.
   d. reflection is not possible.

13. The angle of incidence
   a. must equal the angle of refraction.
   b. is always less than the angle of refraction.
   c. is always greater than the angle of refraction.
   d. may be greater than, less than, or equal to the angle of refraction.

14. A ray of light, which is traveling in air, is incident on a glass plate at a $45$ $\theta$ angle. The angle of refraction in the glass
   a. is less than $45$ $\theta$.
   b. is greater than $45$ $\theta$.
   c. is equal to $45$ $\theta$.
   d. could be any of the above; it all depends on the index of refraction of glass.

15. An oil layer that is 5.0 cm thick is spread smoothly and evenly over the surface of water on a windless day. What is the angle of refraction in the water for a ray of light that has an angle of incidence of $45$ $\theta$ as it enters the oil from the air above? (The index of refraction for oil is 1.15, and for water it is 1.33.)
   a. $27$ $\theta$
   b. $32$ $\theta$
   c. $36$ $\theta$
   d. $39$ $\theta$

16. Lucite has an index of refraction of 1.50. What is its critical angle of incidence?
   a. $1.16$ $\theta$
   b. $15$ $\theta$
   c. $41.8$ $\theta$
   d. $87.4$ $\theta$
17. The critical angle for a substance is measured at 53.7 °. Light enters from air at 45.0 °. At what angle will it continue?
   a. 34.7 °
   b. 45.0 °
   c. 53.7 °
   d. It will not continue, but be totally reflected.

18. Light enters a substance from air at 30.0 ° to the normal. It continues through the substance at 23.0 ° to the normal. What would be the critical angle for this substance?
   a. 53 °
   b. 51.4 °
   c. 36.7 °
   d. 12.6 °

19. A convex lens has focal length f. An object is placed at 2f on the axis. The image formed is located
   a. at 2f.
   b. between f and 2f.
   c. at f.
   d. between the lens and f.

20. A convex lens has a focal length f. An object is placed between f and 2f on the axis. The image formed is located
   a. at 2f.
   b. between f and 2f.
   c. at f.
   d. at a distance greater than 2f from the lens.

21. A convex lens has a focal length f. An object is placed between infinity and 2f from the lens on its axis. The image formed is located
   a. at 2f.
   b. between f and 2f.
   c. at f.
   d. between the lens and f.

22. A convex lens has focal length f. An object is located at infinity. The image formed is located
   a. at 2f.
   b. between f and 2f.
   c. at f.
   d. between the lens and f.

23. A convex lens has a focal length f. An object is placed at f on the axis. The image formed is located
   a. at infinity.
   b. between 2f and infinity.
   c. at 2f.
   d. between f and 2f.

24. An object is placed at 30 cm in front of a diverging lens with a focal length of 10 cm. What is the image distance?
   a. 10 cm
   b. -10 cm
   c. 20 cm
   d. -20 cm
25. An object is placed at a distance of 40 cm from a thin lens. If a virtual images forms at a distance of 50 cm from the lens, on the same side as the object, what is the focal length of the lens?
   a. 45 cm
   b. 75 cm
   c. 90 cm
   d. 200 cm

26. The image of the rare stamp you see through a magnifying glass is
   a. always the same orientation as the stamp.
   b. always upside-down compared to the stamp.
   c. either the same orientation or upside-down, depending on how close the stamp is to the glass.
   d. either the same orientation or upside-down, depending on the thickness of the glass used.
An object 5 centimeters high is placed 30 centimeters from a concave mirror of focal length 10 centimeters as shown above.

(a) On the diagram above, locate the image by tracing two rays that begin at point P and pass through the focal point F. Is the image real or virtual? Is it located to the left or to the right of the mirror?
(b) Calculate the position of the image.
(c) Calculate the size of the image.
(d) Indicate on the diagram above how the ray from point P to point Q is reflected, if aberrations are negligible.
A light ray enters a block of plastic and travels along the path shown above.

(a) By considering the behavior of the ray at point P, determine the speed of light in the plastic.
\[
\sin 37^\circ = \frac{3}{5}, \quad \cos 37^\circ = \frac{4}{5}, \quad \tan 37^\circ = \frac{3}{4}
\]

(b) Determine what will happen to the light ray when it reaches point Q, using the diagram above to illustrate your conclusion.

(c) There is an air bubble in the plastic block that happens to be shaped like a plano-convex lens as shown below. Sketch what happens to parallel rays of light that strike this air bubble. Explain your reasoning.
7-8. An object is located a distance \(3f/2\) from a thin converging lens of focal length \(f\) as shown in the diagram.

(a) Calculate the position of the image.
(b) Trace two of the principal rays to verify the position of the image.
(c) Suppose the object remains fixed and the lens is removed. Another converging lens of focal length \(f_2\) is placed in exactly the same position as the first lens. A new real image larger than the first is now formed. Must the focal length of the second lens be greater or less than \(f\)? Justify your answer.
7-5. An object 4 centimeters high is placed 30 centimeters from a concave mirror of focal length 10 centimeters as shown above.

(a) On the diagram above, locate the image by tracing two rays that begin at point P and pass through the focal point F. Is the image real or virtual? Is it located to the left or to the right of the mirror?
(b) Calculate the position of the image.
(c) Calculate the size of the image.
(d) Indicate on the diagram above how the ray from point P to point Q is reflected, if aberrations are negligible.

a) \( \text{Real, left} \)

b) \[ \frac{1}{f} = \frac{1}{D_o} + \frac{1}{D_i} \]

\[ \frac{1}{f} = \frac{1}{30} \]

\[ \frac{1}{10} = \frac{1}{D_i} \]

\[ \frac{1}{15} = \frac{1}{D_i} \]

\[ D_i = 15 \text{cm} \]

c) \[ H_i = \frac{D_i}{H_o} \]

\[ H_i = -\frac{H_o D_i}{D_o} \]

\[ H_i = -\frac{6 \text{ cm} \cdot 15 \text{ cm}}{30 \text{ cm}} \]

\[ H_i = -3 \text{ cm} \quad \text{(minus sign means it is inverted)} \]
A light ray enters a block of plastic and travels along the path shown above.

(a) By considering the behavior of the ray at point P, determine the speed of light in the plastic.
\[ \sin 37^\circ = \frac{3}{5}, \cos 37^\circ = \frac{4}{5}, \tan 37^\circ = \frac{3}{4} \]

(b) Determine what will happen to the light ray when it reaches point Q, using the diagram above to illustrate your conclusion.

(c) There is an air bubble in the plastic block that happens to be shaped like a plano-convex lens as shown below. Sketch what happens to parallel rays of light that strike this air bubble. Explain your reasoning.

\[ \begin{align*}
1) \quad \eta_1 \sin \theta_1 &= \eta_2 \sin \theta_2 \\
\sin 53^\circ &= \eta_2 (\cos 16^\circ) \\
1,23 &= \eta_2 \\
133 &= \frac{300 \text{ km}}{N} \\
&= 226 \text{ nm/s}
\end{align*} \]

2) \[ \eta_1 \sin 53^\circ = \eta_2 \sin \theta_2 \\
1,06 &= \cos \theta_2 \\
\text{CAN'T DO.}
\]
An object is located a distance $3f/2$ from a thin converging lens of focal length $f$ as shown in the diagram below.

(a) Calculate the position of the image.

(b) Trace two of the principal rays to verify the position of the image.

(c) Suppose the object remains fixed and the lens is removed. Another converging lens of focal length $f_2$ is placed in exactly the same position as the first lens. A new real image larger than the first is now formed. Must the focal length of the second lens be greater or less than $f$? Justify your answer.

\[ p = \frac{3f}{2}, \quad \frac{1}{f} = \frac{1}{p} + \frac{1}{q} \]

\[ \frac{1}{f} = \frac{2f}{3f} + \frac{1}{q} \]

\[ \frac{1}{3f} = \frac{1}{q} \]

\[ q = 3f \]

\[ M = \frac{h_i}{h_o} = -\frac{q}{p} \]

\[ h_i = \frac{a}{q} \]

\[ h_i = 2h_o \]

See appendices.

(c) If the image is larger, the object must be closer to the focus. This means the focal length is larger.

\[ \varepsilon > 0 \quad \text{if} \quad \varepsilon > 3f \]