Torque ($\tau$): The tendency of a force to rotate an object about an axis.

$$\tau = F \cdot d \sin \theta$$

Sometimes, it is easier to use the perpendicular projection of the force rather than the actual force:

If you have time, use trig to prove to yourself that the two situations yield identical results for torque!
Static Equilibrium

So far, we have solved equilibrium problems only by setting horizontal forces: $\Sigma F_x = m\ddot{x}$ and vertical forces: $\Sigma F_y = m\ddot{y}$

$\Sigma F_y = 0$

These two equations don't fully describe the action. Since objects in static equilibrium aren't moving, we can also say that the sum of torques about any arbitrarily chosen axis equals zero:

$\Sigma \tau = I\ddot{\theta} = 0$

Static Equilibrium, therefore, is described by three conditions:

$\Sigma F_x = 0$ (lefts = rights)

$\Sigma F_y = 0$ (ups = downs)

$\Sigma \tau = 0$ (clockwise = counterclockwise)

Example—Pushing on a table:

\[ \begin{align*}
\Sigma F_x &= 0 \\
\Sigma F_y &= 0 \\
\Sigma \tau &= 0
\end{align*} \quad \text{No movement} \]

\[ \begin{align*}
\Sigma F_x &= 0 \\
\Sigma F_y &= 0 \\
\Sigma \tau &= \text{clockwise}
\end{align*} \quad \text{Clockwise rotation} \]

\[ \begin{align*}
\Sigma F_x &= 0 \\
\Sigma F_y &= 0 \\
\Sigma \tau &= 0
\end{align*} \quad \text{No movement} \]

\[ \begin{align*}
\Sigma F_x &\neq \text{left} \\
\Sigma F_y &= 0 \\
\Sigma \tau &= 0
\end{align*} \quad \text{Accelerates to the left} \]
Example: Seesaw

If a child of mass $m$ sits on one end of a seesaw at a distance of $L$ from the pivot, how far from the pivot should the child's father of mass $3m$ sit?

For static equilibrium, we know:

- $\Sigma F_x = 0$
- $\Sigma F_y = 0$
- $\Sigma T = 0$

Horizontal: There are no horizontal forces, so $\Sigma F_x = 0$ is useless.

Vertical: $3mg - mg - 3mg = 0$  

... But this doesn't tell us what we are looking for.

Torque: By taking the axis of rotation to be at $F_N$, we get

- $\Sigma \tau_{F_N} = 0$
- $mgL - 3mx = 0$
- $L - 3x = 0$
- $x = -\frac{L}{3}$

... The father must sit at a distance of $\frac{L}{3}$ from the center.
Example - A hanging sign

\[ \begin{align*}
\mathbf{F}_x &= 0 \\
\mathbf{F}_y &= 0 \\
\mathbf{\tau} &= 0 \\
\mathbf{F}_x &= \mathbf{F}_x - F_r \cos \theta = 0 \\
\mathbf{F}_y &= \mathbf{F}_y + F_r \sin \theta - \frac{1}{2} mg - \frac{1}{2} mg = 0 \\
\mathbf{\tau} &= \mathbf{\tau} = 0 \text{ about any point you choose!} 
\end{align*} \]

The tricky part of this problem is in figuring out the direction for \( F_{wx} \) (left or right) and \( F_{wy} \) (up or down).

We do this as follows:

\( F_{wx} \) must be right, because there is a component of \( F_r \) to the left.

\( F_{wy} \) must be up. We know this because if we choose our arbitrary axis of rotation to be where the cable attaches to the boom, we have the following torques:

- \( F_r \) - no torque
- \( F_{wx} \) - no torque
- \( \frac{1}{2} mg \) and \( \frac{1}{2} mg \) - counterclockwise

In order to balance torque, \( F_{wy} \) must be up to provide a clockwise torque.
Example - Ladder

Assume wall is frictionless, but ground is not!

We figure out reaction forces as follows:

1. \( mg \) (down) must be balanced so that \( \Sigma F_y = 0 \). Since the wall is frictionless, the only other option is the normal force from the ground \( F_{gy} \).

2. By themselves, \( mg \) and \( F_{gy} \) would tend to rotate the ladder clockwise. In order to counter the rotation, we know there must be some normal force from the wall \( F_{wx} \).

3. By itself, \( F_{wx} \) would cause the ladder to accelerate away from the wall. It must be balanced horizontally by friction force from the ground, \( F_{gx} \).

\[ \therefore \text{Using this picture, we have completely described the forces on the system.} \]
3 equations - Note that all statics problems could potentially result in 3 equations and 3 unknowns:

\[ \varepsilon F_x = 0 \]
\[ \varepsilon F_y = 0 \]
\[ \varepsilon I = 0 \]

However, you are more likely to see problems where one of the equations is trivial, for example, the forces on a diving board:

**Example - Diving Board**

A diver has a mass of 50 kg. The board has a mass of 100 kg. If the board is 8 m long, is pinned at the back end (free to rotate, but not move up or down), and is supported by pylons spaced 2 m apart, find the reaction forces \( F_L \) and \( F_R \) in the left and right supports.

\[ \varepsilon F_x : \text{Trivial. There are no horizontal forces} \]

\[ \varepsilon F_y : \quad F_R - F_L - (100)(9.8)(8) = 0 \]
\[ F_R - F_L = 1470 \text{ N} \]

\[ \varepsilon I : \text{Arbitrarily choosing our axis to be at } F_L, \text{ we get} \]
\[ -F_R(2) + (100)(9.8)(4) + (50)(9.8)(6) = 0 \]
\[ F_R = 21560 \text{ N} \]
\[ F_r = 21560 \text{N} \]

Substituting this value for \( F_r \) back into our \( EF_y \) equation:

\[ 21560 \text{N} - F_L = 1470 \text{N} \]

\[ F_L = -20,090 \text{N} \]

\[ \therefore \text{The left support holds the board down with a force of 20,090N, and the right support holds the board up with a force of 21,560N.} \]