Capacitors

We have seen that it is possible to hold two metal plates close together, but not quite touching, such that a positive charge can be stored on one plate and an equal negative charge is stored on another:

If we disconnect the battery, we will be left with two charged plates that can be used to zap things. This apparatus is called a capacitor.

A capacitor's circuit symbol is two parallel lines, with wires coming from each of them:

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Capacitor
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Capacitance

We can see that if we use larger plates (more surface area), we will have more ability to hold charge. Likewise, the closer we space the plates together, the stronger the field between them, and thus the more they can hold charge.

For two parallel plates, charge held \( \propto \) voltage:

\[ Q \propto V \]

For a specific capacitor:

\[ Q = CV \]

Where \( C \) is the "capacitance" of the capacitor:

\[ C \] is the ratio of \( \frac{Q}{V} \) for a given voltage.

\[ C = \text{capacitance } \left[ \frac{\text{Coulomb}}{\text{Volt}} \right] = [\text{Farads or F}] \]

And

\[ C = \frac{\varepsilon_0 A}{d} \]

\( A \) = Cross-sectional area \( [\text{m}^2] \)
\( d \) = Distance between plates \( [\text{m}] \)
\( \varepsilon_0 \) = Permittivity of a vacuum

\[ = 8.85 \cdot 10^{-12} \frac{\text{C}^2}{\text{Nm}^2} \]

*Remember that Coulomb's Constant \( k = \frac{1}{4\pi \varepsilon_0} \)
Dielectrics

If we just used a vacuum between the two plates of a capacitor, we would find it very difficult to make the capacitor small enough to be useful. To solve this problem we use a thin, insulating layer between the plates, called a "dielectric." This solves two problems:

1. The dielectric allows the two plates to be very close together without touching.

\[ C \propto \frac{A}{d} \]

small \( d \rightarrow \) big \( C \)

2. The dielectric actually has an increased permittivity over that of a vacuum. This factor is called \( K \), and it depends on the material chosen:

\[ K_{\text{vacuum}} = 1.000 \]
\[ K_{\text{air}} = 1.0006 \]
\[ K_{\text{rubber}} = 2.8 \]
\[ K_{\text{water}} = 80 \] ← why don't we always use water between two charged plates?

Capacitance with a dielectric becomes:

\[ C = K \epsilon_0 \frac{A}{d} \]

where permittivity of the material (\( \epsilon \)) is:

\[ \epsilon = K \epsilon_0 \]
Dielectric Breakdown (aka "Gapping" or "Arcing")

If too much voltage is applied to a given capacitor, too much charge will build up, and the excess charge will find its way through the dielectric to the other plate. This is called dielectric breakdown, and can permanently destroy a capacitor.

On stormy days, we see this occasionally as the voltage builds up between the clouds and the ground, resulting in the dielectric breakdown of the air between them, also known as lightning.

Energy Stored in a Capacitor

Because it takes work to store charges on a plate, we know that there must be energy stored. This energy is:

\[ U_{\text{electrical}} = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{V} \]