Resistors in Series - A single path for current

\[ I = I_T = I_1 = I_2 = I_3 \] (Cons. of Charge)

\[
\begin{align*}
V_T &= I_T R_T \\
V_1 &= I_1 R_1 \\
V_2 &= I_2 R_2 \\
V_3 &= I_3 R_3
\end{align*}
\] (Ohm's Law)

\[ V_T = V_1 + V_2 + V_3 \] (Cons. of Energy)

By combining the above equations:

\[ I R_T = I R_1 + I R_2 + I R_3 \]

\[ R_T = R_1 + R_2 + R_3 \]

Total resistance in series

Resistors in Parallel - Multiple paths with common start and end

\[ V = V_T = V_1 = V_2 = V_3 \] Cons of Energy

\[
\begin{align*}
V_T &= I_T R_T \\
V_1 &= I_1 R_1 \\
V_2 &= I_2 R_2 \\
V_3 &= I_3 R_3
\end{align*}
\] (Ohm's Law)

\[ I_T = I_1 + I_2 + I_3 \] (Cons of Charge)

\[ \frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \]

\[ \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \]

Total resistance in parallel
Example: Simple Series

Given 3 resistors and battery below, find all values in table.

Steps
1. Find total resistance of whole circuit \( R_T \) by combining individual resistances into a single resistance.
2. Find total current \( I_T \) through battery using \( V_T = I_T R_T \).
3. Find current, voltage, and power of each individual resistor.

\[ R_T = R_1 + R_2 + R_3 \]
\[ R_T = 2000 \Omega + 400 \Omega + 10000 \Omega \]
\[ R_T = 12400 \Omega \]

\[ I_T = \frac{V_T}{R_T} \]
\[ I_T = \frac{6 \text{V}}{12400 \Omega} \]
\[ I_T = 0.00048 \text{A} \]

3. \( I_T = I_1 = I_2 = I_3 \) (series)
\[ I_1 = 0.00048 \text{A} \]
\[ I_2 = 0.00048 \text{A} \]
\[ I_3 = 0.00048 \text{A} \]

\[ V_1 = I_1 R_1 = (0.00048 \times 2000) = 0.96 \text{V} \] (Ohm's Law)
\[ V_2 = I_2 R_2 = (0.00048 \times 400) = 0.19 \text{V} \]
\[ V_3 = I_3 R_3 = (0.00048 \times 10000) = 4.84 \text{V} \]
P_T = V_T I_T = (6 \times 0.00048) = 0.0029W \quad (P = V \cdot I)

P_1 = V_1 I_1 = (0.96 \times 0.00048) = 0.00046W

P_2 = V_2 I_2 = (0.19 \times 0.00048) = 0.00009W

P_3 = V_3 I_3 = (4.81 \times 0.00048) = 0.00232W

**Compound Circuits**

To solve compound circuits which contain both serial and parallel elements, we follow the same steps as before, but the first step is more complicated: we find the total resistance for the whole circuit \( R_T \) by combining resistors in series and in parallel until they eventually simplify into a single resistance. For example, the circuit below simplifies as follows.

\[ \frac{1}{R_{234}} = \frac{1}{R_2} + \frac{1}{R_{34}} \]

\[ R_{12345} = R_1 + R_{234} + R_5 \]

\[ \therefore R_T = R_{12345} \]
With numbers, the above works out as follows.

\[
\begin{align*}
R_1 &= 1 \Omega \\
R_2 &= 4 \Omega \\
R_3 &= 1 \Omega \\
R_4 &= 3 \Omega \\
R_5 &= 7 \Omega
\end{align*}
\]
\[R_T = ??? \Omega\]

\[
\text{R}_3 \text{ in series with } R_4 \text{ becomes...}
\]
\[R_{34} = R_3 + R_4\]

\[
\text{R}_2 \text{ in parallel with } R_34
\]
\[\frac{1}{R_{234}} = \frac{1}{R_2} + \frac{1}{R_{34}}\]

\[
\text{R}_1, \text{R}_{234}, \text{and } R_5 \text{ in series}
\]
\[R_{12345} = R_1 + R_{234} + R_5\]

\[
\therefore R_T = 10 \Omega
\]

The total resistance for the whole circuit is 10 Ohms.
Example - Fill in table for $R_1$, $I$, $V$, and $P$ for the circuit

$R_1 = 4 \Omega$  $R_4 = 6 \Omega$
$R_2 = 6 \Omega$  $R_5 = 4 \Omega$
$R_3 = 6 \Omega$  $R_6 = 2 \Omega$

Step 1 - Simplify to find $R_T$

$R_{56} = R_5 + R_6$

$\frac{1}{R_{456}} = \frac{1}{R_4} + \frac{1}{R_{56}}$

$\frac{1}{R_{3456}} = \frac{1}{R_3} + \frac{1}{R_{456}}$

$R_{13456} = R_1 + R_2 + R_{3456}$

$R_T = 12 \Omega$
Step 2 - Use Ohm's Law to find $I_T$ through the battery

$$V_T = I_TR_T$$

$$I_T = \frac{V_T}{R_T} = \frac{24V}{12\Omega} = 2A$$

We can now fill out the first three values in the table:

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<th>V</th>
<th>P</th>
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<td>T</td>
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<td>24</td>
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$R_T = 12\Omega$

$P_T = V_TI_T = (24V)(2A) = 48W$ 

Step 3 - Use logic to find remaining values

1. Because $I_T = 2A$ and the current thru $R_T = R_1 = R_2 = 2A$

And, since $V = IR$ 

$$V_1 = I_1R_1 = (2)(4) = 8V$$

$$V_2 = I_2R_2 = (2)(6) = 12V$$

$$P = IV \rightarrow P_1 = 16W \quad P_2 = 24W$$
Because there is a total voltage drop of 24V, and 8V drops across R₁, and 12V drops across R₂, 4Volts must drop across R₃, R₄, and R₅.

\[V_3 = 24 - 8 - 12 = 4V\]
\[V_4 = 24 - 8 - 12 = 4V\]

\[\therefore I_3 = \frac{V_3}{R_3} = \frac{4V}{6\Omega} = 0.67A\]
\[I_4 = \frac{V_4}{R_4} = \frac{4V}{6\Omega} = 0.67A\]

\[\rightarrow \text{And } P = IV\]
\[P_3 = (0.67A)(4V) = 2.67W\]
\[P_4 = (0.67A)(4V) = 2.67W\]
Finally, since 2A of current from $R_1$ branches into three paths, of which 0.67A goes through $R_3$ and 0.67A goes through $R_4$, the remainder must go through $R_5$ and $R_6$.

$I_1 = I_3 + I_4 + I_{56}$

$2A = 0.67A + 0.67A + I_{56}$

$\therefore I_{56} = I_5 = I_6 = 0.67A$

$V = IR$

$V_5 = I_5 R_5 = (0.67A)(4\Omega) = 2.67V$

$V_6 = I_6 R_6 = (0.67A)(2\Omega) = 1.33V$

$P = IV$

$P_5 = (0.67A)(2.67V) = 1.78W$

$P_6 = (0.67A)(1.33V) = 0.89W$
The complete table is...

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<th>I</th>
<th>V</th>
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