Angular Velocity \( \omega \) \( \rightarrow \) \( \text{[radians/second]} \) or \( \text{[degrees/second]} \) or \( \text{[RPM]} \)

\[
\omega = \frac{\text{angle}}{\text{time}}
\]

Linear Velocity for an object traveling in a circle

\[
v = \frac{\text{distance}}{\text{time}} = \frac{\text{circumference}}{\text{time}} = \frac{2\pi r}{T}
\]

where \( r \) is radius
\( T \) is “period” - the amount of time to complete one cycle

Frequency vs Period

\[
T = \text{period} = \text{amount of time for one cycle} \ [s]
\]

\[
f = \text{frequency} = \text{number of cycles in a unit of time} \ [\frac{1}{s}] \text{ or } [\text{Hertz}] = [\text{Hz}]
\]

\[
T = \frac{1}{f}
\]

Centripetal Acceleration

In order to travel in a circle, an object must accelerate inward. This should be evident from our definition of velocity, which is speed and direction. Changing direction = changing \( v = \text{acceleration} \).
For an object to turn right, it must accelerate to the right:

![Diagram showing object turning right with acceleration vector]

But what happens if the car continues to accelerate to the right?

![Diagram showing object circling with various accelerations and directions]

- Going right, accelerating down
- Going down, accelerating left
- Going left, accelerating up
- Going up, accelerating right

Notice that:
- The car is driving in a circle, and...
- The acceleration is always toward the center!

This acceleration is called "center-seeking," or centripetal acceleration, \( \vec{a}_c \).
In order for anything to go in a circle, there must be centripetal acceleration happening. The equation for $a_c$ is

$$a_c = \frac{v^2}{r} \quad (** \text{linear velocity})$$

If we substitute $\frac{2\pi r}{T}$ for $v$, we get a derivation of $a_c = \frac{v^2}{r}$

$$a_c = \left(\frac{2\pi r}{T}\right)^2$$

$$a_c = \frac{4\pi^2 r}{T^2}$$

If we substitute $2\pi rf$ for $v$, we get

$$a_c = \left(\frac{2\pi rf}{T}\right)^2$$

$$a_c = 4\pi^2 rf^2$$

$\therefore$ For a given period of rotation, if we increase $r$, we increase acceleration.

Likewise, if we maintain radius constant, but increase the frequency, acceleration goes up.
Centripetal Force

According to Newton's 2nd Law, if there is a centripetal acceleration happening, there must be a net force happening in the centripetal direction:

\[ \Sigma F_{\text{centripetal}} = ma_c \]

\[ F_c = \frac{mv^2}{r} = \frac{m4\pi^2r}{T^2} = m4\pi^2rf^2 \]

**Example** Consider a cone with a frictionless surface on the inside. A block is placed inside the cone and the whole apparatus is set rotating so that the block doesn't slide up or down.

There are two forces on the block: gravity and normal. Since the block isn't accelerating up or down, we know that

\[ \Sigma F_y = ma \]

\[ \Sigma F_y = 0 \]

\[ F_{Ny} - mg = 0 \]

\[ mg = F_{Ny} \]
Looking at the FBD above, we realize that with just the two forces, the forces $F_{Ny}=mg$ are balanced vertically, but not horizontally! This means there must be some horizontal acceleration due to $F_{Nx}$!

Is the block really accelerating horizontally? YES!

This horizontal acceleration is what is causing the block to move in a circle. Thus, the force providing the $a_x$ is the $x$-component of the normal force, $F_{Nx}$: $F_{Nx}=ma_x=\frac{mv^2}{r}$